SKELETON STRUCTURES

APPLICABLE TO

STEEL AND IRON BRIDGES.

HENRICI.
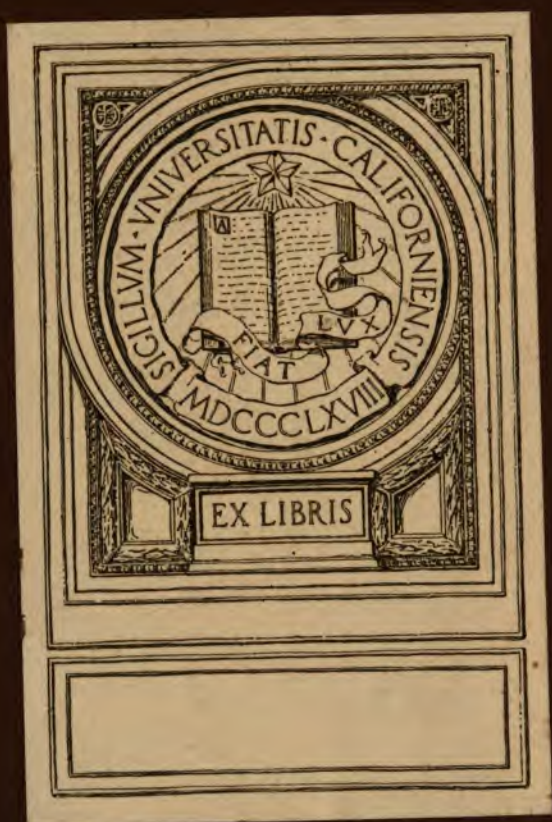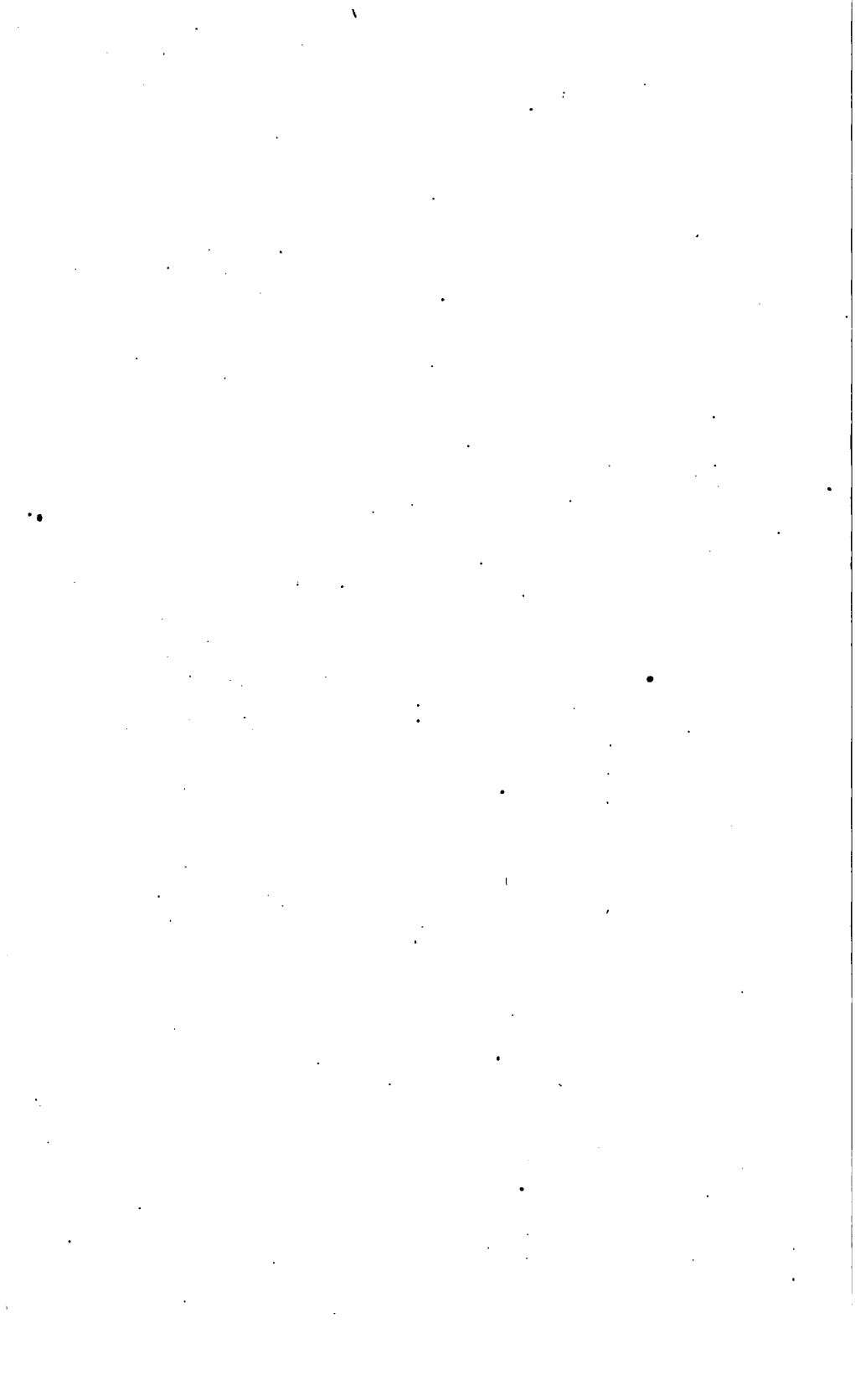
SKELETON STRUCTURES:

ESPECIALLY

IN THEIR APPLICATION TO THE BUILDING

OF

STEEL AND IRON BRIDGES.

BY

OLAUS HENRICI, Ph.D.

WITH FOLDING PLATES AND DIAGRAMS.

PREFACE.

DURING the past year I received from a friend of mine in London some diagrams and descriptions of Suspended and Trussed Bridges, which were part of Mr. Edward Kochs's Patent for "Improvements in the Construction of Beams or Supports applicable to the building of Bridges, Viaducts, Roofs, Arches," etc.

My friend asked me for my opinion, and desired me to examine the matter theoretically.

These designs, as well as those of the remaining part of Mr. Kochs's Patent, agree all in one thing—viz., that all bracing bars which connect the top and bottom tend to two points ; or if one of these points is at an infinite distance, some are parallel. This geometrical design Mr. Kochs seems to have considered very important, and has attached a significance to it which I think it does not deserve, and which an unprejudiced observation will confirm.

Another point, however, which in the said description was clearly expressed, showed that the construction of Bridges, besides this geometrical arrangement, contained a decidedly novel thought, even although Mr. Kochs had not fully recognised its whole importance.

This induced me to look particularly into the matter, and the success has proved the truth of my supposition, and has shown that we can found upon this a new principle of construction, which possesses remarkable advantages : it is the principle which. under the name of Skeleton Structures, is more closely examined in this treatise.

The calculations chiefly carried out for Suspended Bridges have shown particularly that bridges of this system, together with complete rigidity, possess great lightness. Want of time prevented me from completing similar calculations for other structures. I hope, however, to have another opportunity to write on this subject, particularly if the present treatise meets with favor.

By presenting these general examinations on Skeleton Structures, with particular application for Suspended Bridges, to the Engineers of Great Britain —who, at a time when theory in its application to Engineering was very little known, have shown what practical experience and experiments are able to do— I venture to express the hope that they will receive these theoretical results with some confidence, even although an opportunity is wanting to compare them with practical results.

O. H.

Westminster, March, 1866.

SKELETON STRUCTURES.

THE principle of what is here called " Skeleton Struc-
tures," and by which these are distinguished from
others, consists in the manner in which the separate
parts of the structure are united into a whole. In all
these plans the structures are composed of elementary
bars, which are connected at their extremities by round
bolts. Looking, for example, at the beam, Fig. 1, we

Fig. 1.
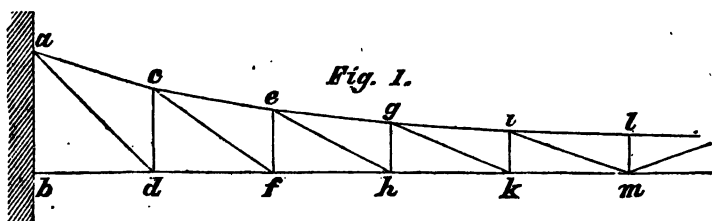
find that each line from connection-point (a, c, d, etc.) to
connection-point represents an independent bar, and
at each end there is a screw bolt, which connects these
bars. A glance at the accompanying tables will make
this clear. No bar reaches farther than from one con-
nection-point to the other, so that we never find any
continuous beams in these structures.

This mode of construction has different advantages ;

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and some of these are of so peculiar a nature, that these structures will soon come into more general use. The principal advantages are :

1. *That each bar is exposed to tension and compression only in the direction of its longitudinal axis*, which is the most favorable condition possible.

2. *All tensile and compressive strains which the different bars are to resist, can be fixed by calculation with the utmost exactness, more so than in other structures.*

By this system the material can be disposed of in the most advantageous manner, and thus be reduced to a minimum. For in no part is it necessary to heap up unused material.

3. *The expenses for making, sending out, and re-erecting are far below those for plate, lattice, and tubular bridges.*

The separate parts consist of bars, steel or iron of suitable section; and the necessary number of bars for resisting the respective strains is introduced. There are no heavy parts in these structures; very little scaffolding is wanted; and all the parts can be finished in the factory, as no riveting is necessary for the re-erection.

4. *Hence it is clear, that such structures can be erected on a larger scale than hitherto.*

5. *These structures are equally well fitted for being made from steel as from iron, and better adapted for this metal than other modes of construction.*

The last three advantages will be obvious to every engineer, without looking further into the matter. I think, however, on account of the importance and peculiar mode of connecting these structures, it

would be advisable to say something general about the saving of material, although I shall not give any figures about weights, until we examine the particular structures. I shall firstly call your attention to a beam A B (Fig 2), which is bent by certain forces

Fig. 2.
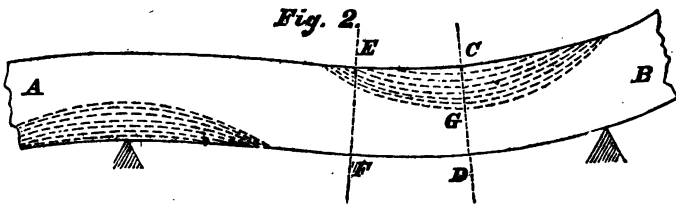
acting upon it. We observe in different points of one section different strains. For example, in the section C D, there is a layer of fibres at G, which is not exposed to any strain (the dotted lines indicate compression) ; because all points above are in compression, and all points below are in tension, and indeed the forces increase with the distance from the fibre G.

Besides this, each fibre is exposed to different strains, as the top fibre, for example, is more in compression at the point C, where the beam is mostly bent, than at E; and as the tension is greater at D than at F, the strains must alter from section to section. To ascertain, therefore, if a beam of this description has sufficient strength, we must find the section of the greatest bending, and in this the fibre of greatest strain ; thus we find the weak point, which has to resist the greatest strain in the whole beam. Does this strain not exceed the amount which this

point can carry safely, we say the beam is strong enough. But it shows further, that a beam exposed to certain forces tending to bend it, uses only a small portion of its material to resist these forces, and the remaining material does not act in proportion to its mass. By introducing beams of the proper shape, we can, it is true, decrease the amount of material,* but we never can come up to a uniform distribution of forces in all parts of the material. Thus, for example, in a common plate girder of double **T** section, the material in the place exerts in proportion to the flanges only one third of its strength (by the above reasons and the distance of material from the neutral fibre.)†

On the contrary, if tension and compression act in a bar only in direction of its longitudinal axis, we find that not only each fibre in the whole length of the bar, but also all fibres of the same section, are exposed to a uniform strain. This shows, that material which is exposed simply to tension and compression, acts incomparably in a more favorable manner, than material which is exposed to bending forces ; and we therefore use less material in structures where the single members are exposed only to tension and compression. Nearly all beams act according to their relative strength, and we cannot prevent them being bent in their whole length ; and if we want to construct a girder exposed only to tension and compression, we must seek for a design, in which the separate

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* By these reasons girders are made in top and bottom considerably stronger (single and double **T** shaped), and the material is increased in the most deflected section.

† Laissle & Schübler, Bau der Brückenträger—Stuttgart, 1857.

parts are not liable to be bent. To make, therefore, a girder in such a way, that a deflection in the whole span is possible, we must connect its single parts by a joint; but in doing so, we must take care not to affect its rigidity. I believe that the above described method of constructing girders of elementary bars will prove to be the only one which fulfils these conditions. This shows, that in a girder of this description the nature of the connection renders it possible to distribute the material exactly according to the strain; and the only question is, if we are in a position to distribute the material so, that in no point is there more than what is necessary. To do this, we must firstly be able to ascertain the exact strain which acts in each single bar. I mentioned already that one of the great advantages of these structures is, that the different strains could be ascertained by calculation with the greatest exactness; and it will be my principal duty to show afterwards to every one, though he be no mathematician, that this assertion is true.

Having ascertained the strains in each bar, it is very easy to make the sections of the bars suitable to the strain, and thus to construct girders of equal strength in each section. By the above statements, if we select a suitable arrangement of the bars, it follows that very little material is required; and we shall see afterwards by the figures, that the weights of these girders are far below those of any other—except, of course, chain and wire-rope bridges, which have not sufficient rigidity. But in comparison with these, the girders in question have the advantage, if the bars

are properly arranged, of being completely rigid in themselves.

After having pointed out the very important advantages of these skeleton structures, it is necessary to examine their disadvantages, and especially to answer the question—are these structures in reality sufficiently rigid? This depends, firstly, on the geometrical design, which will be discussed immediately; and further, on the mode of connection itself, against which, most likely, the chief objections will be made.

ON RIDIGITY OF SKELETON STRUCTURES.

For a girder of this construction to have a perfectly fixed position, it is necessary that each connection-point be fixed absolutely by its meeting bars, and that no part of the structure can move while another keeps its position. A moving in this way, for example, is possible, when in the beam Fig. 3 the bar b c is

Fig. 3.
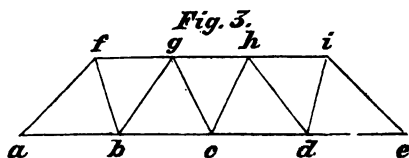
omitted. The part to the right can then turn round g, even although the part to the left keeps its position.; but when the bar b c is replaced, this turning is impossible. Were we to leave out one of the inclined bars, such as g c, no such turning would result, but the parallelogram b g h c can move in its four corners. If, however, g c is replaced, this is not possible. If we fully consider these relations in the most simple geometrical figures, we find that a tri-

angle, which is composed of three bars, connected at
their extremities by screw bolts, is a completely fixed
structure, of which no part can alter its relative
position.

In a four-, five-, or six-sided figure this is *not* the
case. Supposing in a four-sided figure, *a b c d* (Fig.
4) the corner *a* is fixed and *b* likewise—*c* and *d* are
still moveable—but as soon as we
fix *a* and *c* the figure is com-
pletely determined. To fix, there-
fore, the position and figure of a
four-sided figure, we are obliged
to place two opposite corners in
a relatively unchangeable posi-

Fig. 4.
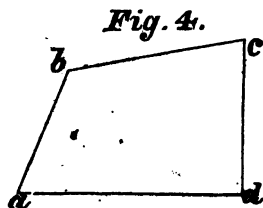
tion. If these points are not determined by the other
part of the structure, the simplest mode of fixing same
is to introduce a diagonal bar *a c.* By introducing,
after this, the second diagonal *b d,* we have one bar
more than is necessary for determining the figure; but
although the second bar gives more rigidity, it has a
disadvantage sufficient to remove all superfluous bars.
For if a superfluous bar has not the exact length, which
it ought to have according to geometrical design, it pro-
duces in the whole structure an additional strain, which
destroys all the advantages of its additional rigidity.
If we introduce, for example, in a rectangular figure
one diagonal, which is rather longer or shorter than it
ought to be, the figure will only assume a slightly
different shape, but will not alter the strain in any of
the bars. But if we introduce a second diagonal, it
is clear that its length must be mathematically accu-
rate, because the figure has been already determined.

If it is a little longer or shorter, it occasions immediately in all the bars a certain tension or compression, which will increase with the inaccuracy of the diagonal. Such additional strains will be produced in every structure *which contains superfluous bars.*

To give an idea of the additional strains caused by the inaccurate length of the bars, we will look at a parallelogram *a b c d* (Fig. 5) in which the diagonal *a c* is of such a length as to cause *a b c d* to be a rectangular figure, and we are to introduce a second diagonal, which is by $\frac{1}{1000}$ part of its length too long.
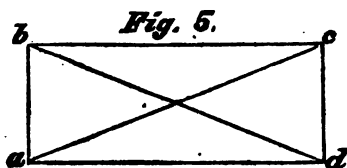
Fig. 5.

The question is now, what is the additional strain caused by this inaccuracy in the length of the bar. It will be found by the afterwards following calculation that the two diagonals are exposed to equal pressure, whilst the sides are exposed to tension. The compression in the diagonal is

$$\tfrac{1}{2} E q \frac{\rho}{r},$$

where E represents the modulus of elasticity, r = the length of bar, q = the section in square inches, and ρ = the additional length of the bar. By taking for steel $E = 29$ millions of lbs. = 12,900 tons, and $\frac{\rho}{r} = \frac{1}{1000}$, the pressure S in the whole section of the diagonals will be = $6\cdot45\ q$ tons and

$$\frac{S}{q} = 6\cdot45 \text{ tons per } \square ''$$

Taking $\frac{\rho}{r} = \frac{1}{10000}$ you will find

$$\frac{S}{q} = 0\cdot65 \text{ tons.}$$

By making a diagonal bar of ten feet in length only $\frac{1}{16}$ of an inch too long ($\frac{1}{32}''$ at each end), it causes an additional compression in each of the two diagonals

$$= \frac{1}{2} \, 12900 \cdot 1 \cdot \frac{1}{120} \cdot \frac{1}{16} \text{ tons.}$$
$$= 3\cdot36 \text{ tons per } \square '' \text{ section.}$$

If this bar is $\frac{1}{16}''$ too short, it causes an additional *tension* of

$$3\cdot36 \text{ tons per } \square ''$$

and this additional material can be saved, if no superfluous bars were used in the construction.

This example also shows clearly that in any structure, if there are more bars than necessary to determine the figure, they are not only superfluous, but injurious and dangerous, because, not knowing the exact amount of error in each, it is not possible to ascertain the exact strain by calculation.

In the skeleton structures we find exactly as many bars as are necessary to make the structure rigid, and where, for particular reasons, it is advisable to introduce superfluous bars, it will be found that the strain caused by the supposed fault in the length of these bars is distributed uniformly through the whole structure, and is therefore in each bar very trifling.

A structure of this kind, by having the necessary number of bars, has the advantage of forming one

single rigid body, the position of which is completely fixed, as soon as three points of it, which are not lying in one straight line, are determined. If the structure is to remain in one plane, which is the case in a common girder, two points will answer this requirement.

The preceding reasonings are only correct on the supposition that there is a perfect connection between the single bars. If the bolts of the connection-points do not quite fill the holes, the points cease to be completely fixed, and allow minute displacings. If there be play only in one of its holes, a triangle also can no longer be regarded as a rigid body, and much less can a larger structure, being a combination of many members. It is, therefore, an essential condition for the permanent strength and solidity of the whole, that the bolts as well as the holes fit exactly; and more care can be bestowed on this point, as this is the only

Fig. 6

work needed to finish the bars. But besides, afterwards, no wearing either of bolts or holes ought to occur. This is the objection which, I have already intimated, would be the most formidable that could be made against these structures.

We will therefore endeavor to discover the means in our power for the practical reduction of this wear. Let A be the head of a bar, B the bolt; the bar is exposed to a
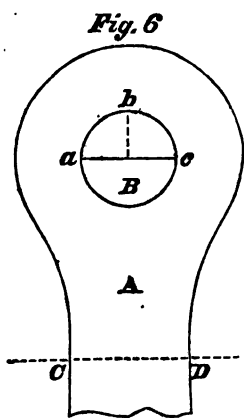
tensile strain, which will be transmitted to the bolt, where it presses against the bar. Through this the bar-head will be subjected to a compression, which in amount is exactly equal to the tensile strain in section $C D$. This force will be distributed over the whole touching-surface, and if the bolt fits well, will extend, but not uniformly, from a to b and c. We will try to ascertain the proportion of this pressure by the following means. A body B (Fig. 7), of a semicircular shape, A B C supports the piece A, the concave surface of which fits it exactly. The pressure is then distributed in such a manner that the section $C D$ is exposed to nearly a uniform compression. If we therefore divide $C D$ into equal parts, each sectional part marked off by such divisions will be exposed to an equal pressure, and this will also be transmitted at the respective pieces of the arch $a\ b\ c$ to the body A. These arch-pieces will become larger towards the points a and c, and will be smallest at b. As the pressure is distributed over the whole touching-surface of each section, the pressure per unit will be largest where the surface is smallest. This is the case at b, and if we take a strake, sufficiently small, its arch will be equal to the corresponding part of the line $C\,D$, therefore the pressure per square inch will here be exactly as strong as on the line $C\,D$; that is, the greatest pressure is at b, and is here of the same amount per square inch as at the line $C\,D$, or in the section $a\ c$.
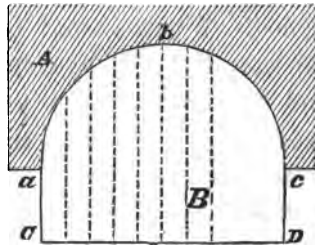
Fig. 7.

Applying this to the bolt, which although not exactly, is sufficiently correct, we find that the total pressure S for the section $a\ B\ c$ (Fig. 6) is exactly equal to the strain in the section $C\ D$. By calculating accordingly the pressure, which acts upon one square inch of the section $a\ B\ c$, we find, as above mentioned, that it is equal to the pressure per $\square''$ at b; $i.\ e.$, equal to the greatest pressure which acts at the touching-surface of bolt and hole.

Now I remind you of the following rule of experience. When a solid body is exposed to a pressure, the material will be exposed to a certain compression, and the greater the pressure the greater the compression. As long as this pressure does not exceed a certain amount, viz., the limit of elasticity in the material, the body will, by virtue of its elastic power, return to its original shape as soon as the pressure ceases to act. But if the pressure exceeds the limit of elasticity, the body will permanently alter its shape. In all cases, therefore, it is necessary to remain under the limit of elasticity, as only then is there sufficient security for the permanent strength of the structure. If the bolts, however, by the pressure alone, are not to suffer any permanent alteration in form, it is necessary that the pressure on the touching-points should in no case exceed the admissible strain of the material. The same can be said of the hole in the bar-head. Is the pressure in no point stronger than is admissible, no enlargement of the hole can ever be caused by the pressure.

Now we have found that the greatest pressure is at b (Fig. 6), and that this is as great per $\square''$ as at the

section $a\,c$. Supposing that the pressure at b is equal to the tension in the bar, it follows that the section $a\,c$ must be equal to the area of the bar. For in this case the pressure per $\square''$ at $C\,D$ is equal to that in the section $a\,B\,c$, and therefore equal to that at b. If the pressure, however, at b be less, the area $a\,c$ must be larger. This section, $a\,c$, represents the longitudinal section of the bolt B (its length being equal to the thickness of the bar), and the area is therefore $= d\,p$, when d is the diameter of the bolt, and p the thickness of the bar-head. If q represents the section $C\,D$, then :

$$d\,p = \text{or} > q.$$

To make $d\,p$ according to this sufficiently strong, we can either take thicker bolts, or enlarge the thickness of the bar-head. If we take, for example, a flat bar of $5'' \times \frac{1}{2}''$ section ; and if the bolt is to be sufficiently strong for shearing, it ought to have a section about equal to that of the bar. The bolt will therefore require a diameter :

$$d = 1\text{·}78 \text{ in}.$$

To keep besides this the pressure at the touching-surface at b below the limit of elasticity, the thickness of the head ought to be at least $1\text{·}5$, because :

$$p\,d = 2\text{·}498,$$
whilst : $q = 2\text{·}5$
making $p\,d = q.$

It will, however, be more advisable to make the diameter d of the bolt stronger, for a thickness of $1\frac{1}{2}''$ in the head on a $\frac{1}{2}''$ bar is objectionable.

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By making the head $1''$ thick, the bolt would require,
at least, a diameter of

$$d = 2\cdot 5''$$

in order to prevent any permanent changes at the touching-surface.

Besides the above described enlargements of the holes, a wearing away would also be produced, when the deflection of the girder causes a slight turning of the bars round the bolts, which gives rise to some friction. Even although this turning of a bar round a bolt is a very slight one, the pressure can be, under circumstances, very considerable ; and a movement will be caused by each passing load. The wearing out is certainly very small indeed, as long as the bolts fit exactly. To reduce, therefore, this friction to a minimum, we need only harden the bolts, as well as the surface of the holes. By this hardening, the power of " *resistance against pressure*" is at the same time greatly increased.

That, therefore, the connection between the bolts and bars may afford sufficient strength and permanent solidity, we are obliged to make the thickness of bolt and bar-head according to these rules.

If further, these bolts fit exactly, and are superficially hardened, there is no reason whatever for this connection wearing more or not having the same duration as any other.

The above discussed points might have found but too little consideration hitherto.

THEORETICAL CALCULATION OF THE STRAINS.

I will now proceed to explain the theoretical principles, by which we are enabled with the expense of time, but without especial mathematical assistance, to ascertain exactly all the strains which act in the single bars of the structure. I shall take particular care to present the matter in such a form, that any one who is acquainted with only the first elements of mathematics, will understand it, or at least will be convinced that the forces can be determined exactly by calculation. As all the results, which are afterwards arrived at, are found by means of calculation, the reader will only then acknowledge the value of the construction when he is convinced of the correctness of the theory.

These inquiries must naturally be based upon rules taken from *experience.* Having thus gained a good basis for the application of the ever-true rules of mathematics, we will find that the results obtained by the aid of mathematical formulæ must give *the same correctness* as the basis.

Experience teaches us, if a bar be exposed to a tensile strain in the direction of its longitudinal axis, it will be slightly prolonged. At the same time the elasticity of the rod comes into action and causes a strain, which acts contrary to the exterior tensile force, and thus produces equilibrium. The exterior tensile force serves us as measure for this strain. In this action, however, the strain in the rod is directly opposed to the tensile force, and tends to stop the

prolongation. This action takes place as soon as we
decrease the tensile force, or remove it entirely ; the
strain within the rod will immediately contract it, and
reduce it to its original length. This, however, is
found by experience to be true only so far as the
prolongation does not exceed a certain amount ; *i. e.,*
as long as the tensile force keeps below the limit of
elasticity.

Thus we find the tensile force for steel at its
limit of elasticity $= 15\cdot6$ tons per ☐ in., which causes
a prolongation of $\frac{1}{808}$ of the original length ; for
iron these figures are $8\cdot3$ tons strain, and $\frac{1}{1530}$ pro-
longation.

Does the force exceed this limit, the rod will be
liable to a *permanent* prolongation, and this force will
destroy a steel bar, if it amounts to $50\cdot4$ tons, and an
iron bar, if it amounts to $22\cdot9$ tons per in. ☐ .

For practical purposes we must always keep below
the limit of elasticity, and the farther, the less homo-
geneous the material is. Thus we generally take as
the admissible strain for iron 4—6 tons per square inch,
and for steel 8—$12\cdot5$ tons, according to the nature of
the structure and the material applied. This number
will also be found by experience. The above figures
apply to ordinary steel—for cast steel the limit of
elasticity is higher, and as this material is even more
homogeneous, the admissible strain can be taken very
near to the limit of elasticity.

These figures, taken from experience, show that
the prolongation of a bar at the highest admissible
strain is, in proportion to its original length, very
small. If, for instance, a load of 12 tons per ☐ in.

is applied to steel, its prolongation will be $= 0.0009$; *i. e.* $= \frac{1}{1078}$ of its length, and for iron it will be found to be far less.

We must now find the relation between the prolongation and the strain ; for when we have expressed this by a mathematical formula, we can calculate the one if we know the other. By direct trials it is ascertained that the prolongation of a rod increases proportionally to the tensile force and to its length, and decreases proportionally to its sectional area. Thus, if a load of 5 tons causes in a rod of 1 ☐ in. section and 1 foot length a prolongation ρ; a load of 10 tons will cause in the same rod a prolongation 2 ρ. The same will take place when a load of 5 tons is applied to a rod of 1 ☐ in. section and 2 feet length ; whilst a rod of 2 ☐ in. section and 1 foot length at 5 tons load extends only $\frac{1}{2}$ ρ.

Practically, we can consider this law perfectly correct for all structures, and we have thus a basis for further reasoning. This expressed by formulæ will be, if r is the length of a bar, q the section, and P a load which causes a prolongation ρ :

$$\rho = \frac{P\,r}{E\,q}, \quad \cdot \quad \cdot \quad \cdot \quad (1)$$

where E, generally called modulus of elasticity, is a quantity, which depends on the nature of the material, and is determined by experiment. The quantity of this modulus is always large. For steel it is $= 29,000,000$ lbs. per ☐ in. ; so that ρ, according to formula (1), becomes, in reality, very small. By applying a compressive instead of a tensile force, we have, instead of

prolongation, a shortening ; and instead of the strain, which tends to contract the rod to its original length, we have a pressure in a contrary direction.

As now the interior and exterior tensile and compressive forces, as likewise the prolongation and shortening, are contrary to each other, we will use the same symbols, but with reverse signs.   Therefore $-P$ is a compressive force, $-S$ the corresponding pressure within the rod, and $-\rho$ the shortening.

We know from experience that we can use for compression the same rule as for tension.   It is :

$$-\rho = -\frac{P\,r}{E\,q}$$

or if on each side the signs are changed :

$$\rho = \frac{P\,r}{E\,q}$$

For most of the materials however the limit within which the formulæ are right is different for tension and compression ; but as we in practice always keep within these limits, we can consider formula (1) to be for general use quite exact.

In the formula, P represents the exterior compressive force, and as this one is equal to the tension acting in the bar, we may substitute for P the tension S, and thus get :

$$\rho = \frac{S\,r}{E\,q}, \qquad . \qquad . \qquad \text{(2)}$$

which makes :

$$S = \frac{E\,q\,\rho}{r}. \qquad . \qquad . \qquad \text{(3)}$$

So that, as soon as we know the prolongation, we can calculate the strain, which will be tension if ρ is positive, and compression if ρ is negative.

By the aid of these formulæ (2) and (3), we are able to solve a number of problems, as I shall show when I have explained their general character. These problems are all contained in the principal one—to calculate the forces which act in any system of bars, bolted together as previously described. I consider here only such systems as lie in one plane, which most girders do.

If a force act upon a material point, it will cause this to move, if no other forces act in a contrary direction.* Should, therefore, a point exposed to several forces remain at rest, the resulting force (that is, the force composed of all the other forces, according to the "parallelogram of forces") should disappear, for otherwise this resultant would put the point in motion.

On a connection-point act, firstly, the strains of the different bars, which run together here, and most likely also an exterior load which rests on the bolt. The strains in the bars assume therefore such values, that the resulting force at each connection-point will disappear.

Should this not be the case, the resulting force will move the bolt in its direction. . By this the length, and, according to the previous example, also the strains of all bars become slightly altered, and there-

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* The elasticity of material may also be called such a force.

fore the displacings will continue so long as the points rest in equilibrium.

Into such a position each point of a structure will enter, and will remain there as long as the exterior forces continue to act.

We are now to determine the strains in a structure which is in equilibrium. To make it clearer we will look at a bridge-girder. The acting (or rolling) load will cause the girder to deflect, and therefore all the connection-points become displaced. These displacings expand and contract the different bars, and cause such strains as mutually preserve equilibrium. To determine these strains we will use the above rule, which may be explained as follows :

In each structure composed in the above way the resulting force of the strains in the bars and the exterior forces will in each connection-point be = 0.

This rule expressed by mathematics gives for each structure always exactly·so many equations as are necessary for ·the determination of all the strains. And these equations being of the first degree will not cause any difficulty in their solution except by their number.

Before proceeding with the calculation of simple examples, I will you remind that the already mentioned displacings of the connexion-points are always very trifling. As a small alteration in the length of a bar is sufficient to cause a great strain, small displacings will also suffice to produce such strains as are requisite to preserve equilibrium under the acting load. By no means must these displacings assume so great a value that the prolongation of a bar would cause a

greater strain than the material employed would allow.

To express the above rule in formulæ we will adopt a different mode of resolving the strains than is generally done in the parallelogram of forces. By this mode, as soon as we know all the forces in direction and magnitude, we are able to represent any number of forces acting upon a point by a single force. It enables us further to resolve any force into two given directions, and to ascertain the amount of these two forces ; so that if, besides a number of known forces, also two unknown ones of given direction are acting in one point, we are able to arrange these two forces in such a manner that the resulting force of all disappears. If more than two forces be unknown, the parallelogram of forces is insufficient, and we are obliged to use calculation.

If several forces are acting upon one point, we are able to divide each of them into two of given direction. Let, for example, in Fig. 8, *O A* represent in length and direction a force acting upon *O* ; we are able to resolve this one into two, perpendicular to each other, *O X* and *O Y*, and thus we call the corresponding forces *O C* and *O B* the *com-*

Fig. 8.

ponent forces, which can be ascertained after the direction of *O X* and *O Y* is fixed.
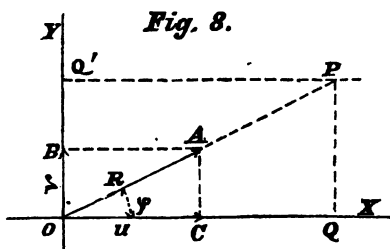
Because △ *O C A* is rectangular, we find :

$$R^2 = U^2 + V^2$$

and secondly, that the proportion of U and V to R is determined, as soon as the direction of R to $O\ C$ is known. Taking now in the line $O\ A$, which represents the 'direction of the force R, any point P, and drawing lines from it vertical to $O\ X$ and $O\ Y$, viz. : $P\ Q$ and $P\ Q'$, we find :

$$\Delta\ O\ C\ A \backsim O\ Q\ P,$$
$$\Delta\ O\ B\ A \backsim C\ Q'\ P,$$

and therefore :

$$O\ C : O\ A = O\ Q : O\ P$$

or :

$$U : R = O\ Q : O\ P,$$

and likewise :

$$V : R = O\ Q' : O\ P.$$

By this it follows that

$$U = \frac{O\ Q}{O\ P}.\ R,$$

$$V = \frac{O\ Q'}{O\ P}.\ R$$

or :

$$U = R\ \cos\ \phi,$$
$$V = R\ \sin\ \phi,$$

because

$$\frac{O\ Q}{O\ P} = \cos\ \phi$$

and

$$\frac{O\ Q'}{O\ P} = \sin\ \phi.$$

We see by this that we always are able to ascertain U and V as soon as the amount and direction of R towards $O\,X$ and $O\,Y$ are given.

Taking a point upon which any number of forces are acting, we may draw two lines (axes) vertical to each other through this point, and can thus resolve each force (R) into its component forces $(U$ and $V)$. Thus we get, instead of forces of various directions, forces in the direction of their verticle axes; which, like all forces of the same direction, can be added or subtracted according to their positive or negative value. After having done this, we have in each axis only one force, and by combining these we find the resulting force of the whole system. *This resulting force must be in each connexion-point* = 0. Supposing the two last component forces are U' and V' we have, according to our formula, for the resulting force R'

$$R'^2 = U'^2 + V'^2$$

and we see by this, that if R' shall disappear, U' and V' must be = 0; because the sum of two positive figures (like U'^2 and V'^2), can only disappear if both numbers by themselves become nought.

A glance at Fig. 9 will show this plainly, the diagonal R' in the rectangle can only be equal nought, if both sides U' and V' disappear. This leads us to the conclusion :

Fig. 9.
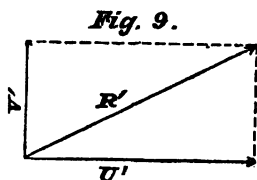
That in each connexion-point both components of the resulting force (considered in reference to any two vertical axes) must disappear.

Thus we get from the one condition, that the resulting force must disappear, two conditions, and therefore two equations, between the strains.

EXAMPLES.
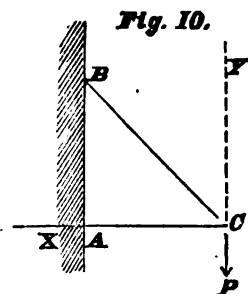
Fig. 10. In the structure Fig. 10, $A\ C$ and $B\ C$ represent bars, which are connected at A and B by bolts to a wall piece, and at C by bolt to each other. A weight P acts on the bolt C, and we want to ascertain the strain in the two bars. Firstly we have to apply two rectangular axes through C, and because $A\ C$ is horizontal, and $C\ P$ vertical, we take these directions as the axes. The strain in the bar $B\ C$ we call S, and the one in $A\ C$, T. If S is resolved according to the axes, into its components, we find

$$\text{in the direction of } C\,X : S\,\frac{A\,C}{B\,C},$$

$$`` \qquad `` \qquad C\,Y : S\,\frac{A\,B}{B\,C}.$$

The two other forces act in the direction of the axes; hence we get

$$\text{in the direction of } C\,X : S\,\frac{A\,B}{B\,C} + T,$$

$$`` \qquad `` \qquad C\,Y : S\,\frac{A\,B}{B\,C} - P,$$

where P is subtracted, because it acts contrary to the

strain S. Both components must disappear, we have therefore :

$$S\,\frac{A\,C}{B\,C} + T = 0,$$

$$S\,\frac{A\,B}{B\,C} - P = 0.$$

From these equations we see that

$$S = \frac{B\,C}{A\,B}\cdot P,$$

$$T = -\,\frac{A\,C}{A\,B}\cdot P\,;$$

which shows, that in the bar $B\,C$ there is tension, and in $A\,C$ compression, because T is negative.

In this example we had only one connection-point ; we shall take, therefore, a more complicated example by joining more bars to those, and thus we get a structure like the one pre-sented in Fig. 11, of which we shall proceed to calculate the strains. We will call here the connection-points 0, 1, 2, 3, &c., and the bars will be represented by two figures (01), (12), &c. ; the

Fig. 11.

lengths of the bars we call r with the respective figures, r_{13}, &c., and the strains likewise, S_{13}, &c. We take now
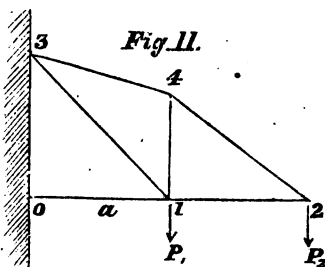
$$r_{01} = r_{12} = a,$$
$$r_{03} = a,$$
$$r_{14} = \frac{3}{4}\,a,$$

and find further :

$$(r_{12})^2 = a^2 + a^2 = 2a^2,$$

$$(r_{34})^2 = a^2 + \left(\frac{a}{4}\right)^2 = a^2 \left(1 + \frac{1}{16}\right)$$

$$= \frac{17}{16} a^2,$$

$$(r_{24})^2 = a^2 + \left(\frac{3a}{4}\right)^2 = a^2 \left(1 + \frac{9}{16}\right)$$

$$= \frac{25}{16} a^2,$$

therefore :

$$r_{12} = a \sqrt{2},$$

$$r_{34} = a \frac{\sqrt{17}}{4},$$

$$r_{24} = \frac{5}{4} a .$$

By this all bars are determined in length and posi-
tion, and we can proceed to resolve the forces. For
axes we always take one horizontal and one vertical
line, and because we have three free connection-points,
1, 2, and 4, we can form six equations, by which we
are able to ascertain all the six strains at once. For
point 2 we have the same equations as in the previous
example, by introducing S_{24} instead of S; and S_{12} in-
stead of T; as likewise r_{14}, r_{12}, and r_{24} instead of
AB, AC, and BC; and we get

$$S_{24} = \frac{r_{24}}{r_{14}} . P_2 ,$$

$$S_{12} = - \frac{r_{12}}{r_{14}} . P_2 ;$$

or by introducing the values of r :

$$S_{24} = \frac{5}{3} P_2 , \ .$$

$$S_{12} = - \frac{4}{3} P_2 . \ .$$

Looking at point 4, we get :

S_{24} resolved into $S_{24} \cdot \dfrac{r_{12}}{r_{24}}$ and $S_{24} \cdot \dfrac{r_{14}}{r_{24}}$,

S_{34} "   " $S_{34} \cdot \dfrac{r_{01}}{r_{34}}$ " $S_{34} \cdot \dfrac{r_{03} - r_{14}}{r_{34}}$

S_{14} "   "   0   " $S_{14} .$

By adding these, we get 2 equations :

$$S_{24} \frac{r_{12}}{r_{24}} - S_{34} \frac{r_{01}}{r_{34}} = 0,$$

$$S_{34} \frac{r_{14}}{r_{24}} - S_{34} \frac{r_{03} - r_{24}}{r_{34}} + S_{14} = 0 .$$

The first equation will give

$$S_{34} = \frac{r_{34}}{r_{01}} \cdot \frac{r_{12}}{r_{24}} \cdot S_{24} ,$$

or because

$$S_{24} = \frac{5}{3} P_2 \ \text{and} \ \frac{r_{34}}{r_{01}} \cdot \frac{r_{12}}{r_{24}} = \frac{\sqrt{17}}{5},$$

$$S_{34} = \frac{\sqrt{17}}{3} P_2 .$$

From the other equation follows

$$S_{14} = - \frac{2}{3} P_2 .$$

We have still to ascertain the strains S_{01} and S_{13}, which we are able to do by the equations of point (1), after having altered them in the same way:

$$S_{01} + \frac{r_{01}}{r_{13}} \cdot S_{13} - S_{12} = 0 \, ,$$

$$S_{14} + \frac{r_{02}}{r_{12}} \cdot S_{13} = P_1 \, ,$$

By which, in connection with the above equations, we find

$$S_{13} = \sqrt{2} \, P_1 - \frac{3}{4} \sqrt{2} \cdot P_2 \, .$$

$$S_{01} = - \, P_1 - \frac{7}{12} P_2 \, .$$

It will be clearly understood how these calculations are carried on. Taking a structure like that in Fig. 12 the calculations are done in the same way; one strain is calculated after the other without difficulty.
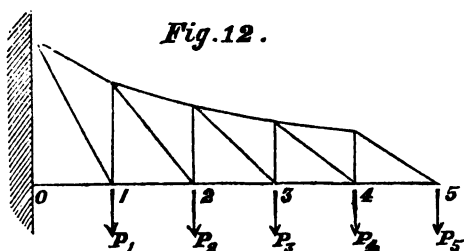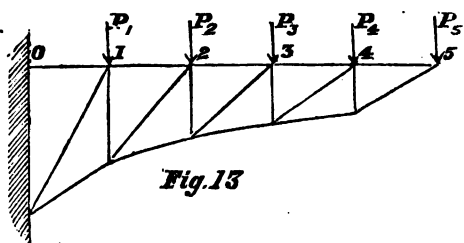
Fig.12.

I will mention here at once, that the same calculation can be used for a trussed structure, as in Fig. 13; the structure is exactly the reverse of Fig. 12, and it is

Fig.13

not necessary to alter the formulæ; for as all the Ps

act here in the contrary direction, all the signs will be reversed. All the strains, therefore, will be the reverse of those in Fig. 12 ; where there was tension in a bar there is now compression ; and where there was compression, tension.

In these examples, we had always exactly as many bars in the structure as was necessary to give it rigidity, and nowhere had we any superfluous bars. (*See* page 12.) In all cases where this is fulfilled it is possible to ascertain the strains in the single bars by the mode above described.

As we have shown that the strains can be calculated very exactly (and the sections of the bars made accordingly), we have *proved one of the former asser-tions*, viz., that each bar can be made exactly as strong as necessary ; and thus we obtain a structure which under a certain load *shows in all parts the same strain per square inch.*

In all structures in which we find superfluous bars, or in all those in which more points are fixed than is necessary to fix the structure, we are obliged to ascertain the various strains in a different way.

In all structures without superfluous bars we have, as shown above, exactly as many equations as there are bars ; or, by including the fixed points of con-nexion, and by adding the pressure of the abutments, we obtain exactly as many equations as there are strains to be calculated.

The number of equations depends on the number

3

of connection-points ; each one gives two equations,
and this number is not changed by the omission or
introduction of some bars (though the omission would
affect the rigidity of the structure). Further, as we
are able to calculate the same number of strains as we
have equations, we would want for each superfluous
bar *one* equation. I shall show hereafter that the
above number of equations is still sufficient to deter-
mine all the strains.

In the beginning of this part I mentioned that
each girder deflects under any load, and that each con-
nection-point would be displaced, but that these dis-
placings would be very trifling. Now we are able to
resolve these displacings in the same way as has been
done above with the forces, into two displacings per-
pendicular to each other. Has any displacing of this
kind caused the point O (Fig. 14) to
go to A, displacings can be present-
ed by $O\ B$ and $B\ A$, or by $O\ B$ and
$O\ C$; because $O\ C = B\ A$, which
represent the displacing in the same
way as the components above represented the force.
If therefore the two component displacings are known,
we know also the actual displacing in magnitude and
direction.
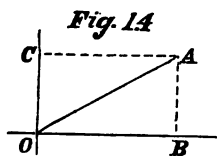
Having ascertained the displacing at each end of
each bar, *i. e.* the displacing of two connection-points,
we know also the exact length of the bar in this state ;
and by deducting the original length we get a small
piece ρ, which is the prolongation of the bar.

*As soon as, therefore, we know the displacing of all the
connection-points we know the prolongations of all the
bars ;* and I mentioned at the beginning, *that the strain*

*in any bar could be ascertained by the amount of the pro-
longation.* The equation was :

$$S = \frac{E\,q\,\rho}{r}.$$

This shows that we are able to calculate all strains
as soon as the displacings of all connection-points are
known. We have, therefore, now to ascertain the
displacings of these points, and will look for this pur-
pose to Fig. 15. Let i and k represent two of these
points, which are connected by a bar ($i\,k$). After the
structure is loaded, we assume this bar has moved from
the position ($i\,k$) to ($i'k'$), and ii' and kk' are accord-
ingly the displacings of the connection-points. Let
$O\,X$ and $O\,Y$ be the axes, horizontal and vertical,
then $i\,a$ and $i'a$ are the projections of the displacing
ii' in the direction of these axes. Let

$$i'a = u_i, \qquad i\,a = v_i,$$

and let the displacings of point k, viz., $k'd$ and $k\,d$,
be respectively $= u_k$ and v_k. If, further, r represent

Fig. 15.
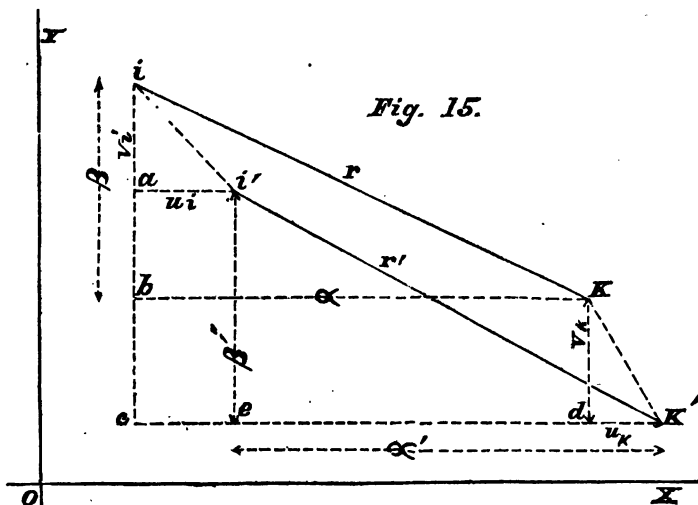
the length $i\,k$ and r' that one of $i'\,k'$; then is,

$$r' - r = \rho,$$

i. e. the prolongation of the bar at this displacing ; therefore

$$r' = r + \rho.$$

Let us put

$$i\,b = \beta\,, \qquad\qquad k\,b = a\,,$$
$$i'\,e = \beta',\qquad\qquad k'\,e = a',$$

then is :

$$r^2 = a^2 + \beta^2, \qquad\qquad (r')^2 = (a')^2 + (\beta')^2\ ;$$

further, as the figure shows clearly,

$$a' = e\,k' = c\,k' - a\,i' = b\,k + d\,k' - a\,i';$$

or taking the above notations,

$$a' = a + u_k - u_i\,;$$

likewise

$$\text{\textbackslash}\quad \beta' = \beta + v_k - v_i\,.$$

Now we had

$$(r')^2 = (a')^2 + (\beta')^2$$

and by putting $r + \rho$ instead of r', and by introducing instead of a' and β' their values, we find :

$$(r + \rho)^2 = (a + u_k + u_i)^2 + (\beta + v_k - v_i)^2$$

or, solved :

$$r^2 + 2\,r\,\rho + \rho^2 = a^2 + 2\,a\,(u_k - u_i) + (u_k - u_i)^2$$
$$+ \beta^2 + 2\,\beta\,(v_k + v_i) + (v_k - v_i)^2.$$

And because $\qquad r^2 = a^2 + \beta^2,$

we get by subtraction:

$$2 \, r \, \rho + \rho^2 = 2 \, a \, (u_k - u_i) + 2 \, \beta \, (v_k - v_i)$$
$$+ (u_k - u_i)^2 + (v_k - v_i)^2 \quad \ldots \quad (4)$$

Now we have shown before, that ρ is always a very small fraction; therefore ρ^2 is remarkably small, and even in comparison with ρ itself, so small, that we may omit it altogether. The same may be done with the square of $u_k - u_i$ and with $v_k - v_i$. By this omission it follows from the above equations:

$$2 \, r \, \rho = 2 \, a \, (u_k - u_i) + 2 \, \beta \, (v_k - v_i);$$

and this divided by $2 \, r$,

$$\rho = \frac{a \, (u_k - u_i) + \beta \, (v_k - v_i)}{r} \quad \ldots \ldots \quad (5)$$

As now

$$S = \frac{E \, q \, \rho}{r}, \qquad \ldots \ldots \quad (3)$$

we get

$$S = E \, q \, \frac{a \, (u_k - u_i) + \beta \, (v_k - v_i)}{r^2} \ldots \quad (6)$$

According to this a strain is expressed by the displacings in the extremities of the bars; the displacings themselves are, however, still unknown. But we have gained one advantage; we have determined more than $2n$ strains by $2n$ unknown values, where n is the number of connection-points. We have seen before, that each connection-point gives two equations for the strains, and we thus get $2n$ equa-

tions. By introducing, therefore, in these equations the amount of strain expressed by the displacings, we get from these $2n$ displacings the same number of equations, in which no other unknown values are present. These equations are also for the displacings of the first degree, and thus completely sufficient to determine same. Formula (3) gives then the strains.

The equations between the strains (S) were determined by resolving all the forces at one connection-point into its two components, and by causing the sum of all the components to disappear along each one of the axes. By introducing the same axes as in Fig. 15, the components of the strains will be

$$S \frac{a}{r} \text{ and } S \frac{\beta}{r},$$

or expressed by these displacings

$$S \frac{a}{r} = E q a \frac{a (u_k - u_i) + \beta (v_k - v_i)}{r^3{}_{ik}},$$

$$S \frac{\beta}{r} = E q \beta \frac{a (u_k - u_i) + \beta (v_k - v_i)}{r^3{}_{ik}}.$$

By finding these values for each connection-point, and by causing the sum of all components along the same axis to disappear, we find the equations, which in all cases are sufficient to ascertain the displacings of all points.

Thus it is possible by the solution of a system of 1st degree equations to calculate also these strains.

By this calculation we find with the same exactness the *deflection* of a girder, because the vertical displacing of a point is always its deflection.

The exactness in such a calculation of the strains is remarkable. We got before

$$2\,r\,\rho + \rho^2 = 2\,a\,(u_k - u_i) + 2\,\beta\,(v_k - u_i)$$
$$+ (u_k - u_i)^2 + (v_k - v_i)^2 \qquad \ldots (4)$$

and in this we omitted :

$$\rho^2\,;\,(u_k - u_i)^2 \text{ and } (v_k - v_i)^2$$

by making them equal nought. This is, however, not absolutely correct, and we commit an error, which is, however, so small, that it does not affect the result. This is shown, for example, in steel, in which, under a strain of twelve tons per ☐ inch ρ is always smaller or equal to $0\cdot0009.r$ — it can never be more, and will seldom reach this value. For iron, ρ will be found to be smaller in proportion to its admissible strain.

It is not difficult to ascertain the exact amount of this error, because we have in $2\,r\,\rho + \rho^2$ neglected the small value of ρ^2 and put instead of the above value only $2\,r\,\rho$. This divided by $2\,r$ gives.

$$\frac{2\,r\,\rho + \rho^2}{2\,r} = \rho + \frac{\rho^2}{2\,r} = \rho\left(1 + \frac{\rho}{2\,r.}\right)$$

As we have just shown $\frac{\rho}{r}$ can in the highest value only be equal $0\cdot0009$, which makes $\rho\left(1 + \frac{\rho}{2\,r}\right) = 1\cdot00045\,\rho$. By putting for this value simple ρ, we neglect

$$\frac{9}{20000}\rho$$

being not even $\frac{1}{1000}$ part of ρ.

Equally insignificant are $(u_k - u_i)$ and $(v_k - v_i)$ and considering we are neglecting in equation (4) on each side a positive value of nearly the same amount, the fault gets so trifling that it can hardly be called an error at all. If we take into account these small quantities, the calculation becomes remarkably more difficult, whilst the result would be at last exactly the same ; as the greater exactness would only affect the decimals, which are *never* used in practice.

<div align="center">EXAMPLE.</div>
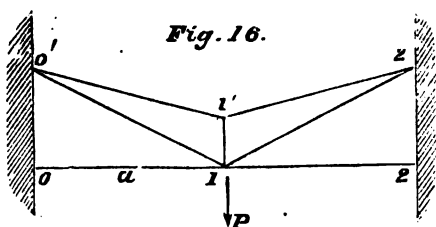
Fig. 16.

It will be seen at once, that in a structure like Fig. 16 we have superfluous bars, because four points are fixed and two points are sufficient to determine the position of the structure ; and further we get only *four* equations for the *two* points (1) and (1'), by the aid of which we have to calculate the strain in *seven* bars. The denotation is the same as before ; the strain in the bar (0 1) is expressed by S_{01}, its length by r_{01}, its section by q_{01} ; and so likewise for the other bars. To determine the lengths, we take

$$r_{01} = r_{12} = a,$$

$$r_{00'} = r_{22'} = \frac{a}{2},$$

$$r_{11'} = \frac{a}{4}.$$

The remaining bars are fixed by these in length, and we find from the "theorem of Pythagoras"

$$(r_{0,1})^2 = (r_{01})^2 + (r_{00'} - r_{11'})^2$$
$$= a^2 + \frac{a}{4} = a^2 \left(1 + \tfrac{1}{16}\right)$$
$$= 1{\cdot}0625 \cdot a^2,$$
$$r_{0,1'} = a \sqrt{1{\cdot}0625} = 1{\cdot}0307 \cdot a.$$

And likewise for the others :

$$r_{0,1} = 1{\cdot}118\ a,$$
$$(r_{0,1})^2 = 1{\cdot}25\ a,$$
$$(r_{0,1})^3 = 1{\cdot}398\ a^3,$$
$$r_{0,1'} = 1{\cdot}031.\ a,$$
$$r_{0,1'}{}^2 = 1{\cdot}0625 \cdot a^2, \qquad r_{0,1'}{}^3 = 1{\cdot}095\ a^3,$$
$$r_{11'} = 0{\cdot}25 \cdot a,$$
$$r_{11'}{}^2 = 0{\cdot}0625 \cdot a^2,$$
$$r_{11'}{}^3 = 0{\cdot}0156 \cdot a^3,$$
$$r_{1,2'} = r_{0,1'},$$
$$r_{12'} = r_{0,1}.$$

The squares and cubes have been added, as they are required in the calculation.

Firstly, we must resolve the forces into their components ; and I will mention at once that, because the figure is symmetrical, the strains to the left of 11′ are equal to those to the right ; i. e.,

$$S_{1,2'} = S_{0,1'}$$
$$S_{12'} = S_{0,1'},$$

and we have to calculate the four strains :

$$S_{0,1}, \ S_{0,1'}, \ S_{11'}, \ \text{and} \ S_{01}.$$

The axes shall be in horizontal and vertical direction ; and resolving according to these, the components of point (1) will be

	Horizontal.	Vertical.
S_{01} :	S_{01} ,	0 ,
$S_{10'}$:	$S_{01'} \dfrac{a}{r_{0'1}}$,	$S_{01'} \dfrac{\frac{1}{2}a}{r_{01'}}$,
$S_{11'}$:	0,	$S_{11'}$,
P :	0,	P,

and for point (1') they will be :

	Horizontal.	Vertical.
$S_{0'1'}$:	$S_{0'1'} \dfrac{a}{r_{0'1'}}$,	$S_{0'1'} \dfrac{\frac{1}{4}a}{r_{0'1'}}$,
$S_{11'}$:	0,	$S_{11'}$.

This enables us to form two equations, because the equations for the horizontal components will only tell us that, for example,

$$S_{0'1'} = S_{1'2'}.$$

There remain, therefore, only the two equations from the vertical components,

$$2S_{0'1} \frac{\frac{1}{2}a}{r_{0'1}} + S_{11'} = P,$$

$$2S_{0'1'} \frac{\frac{1}{4}a}{r_{0'1'}} - S_{11'} = 0,$$

wherein $S_{0'1}$ and $S_{0'1'}$ are taken double, because members equal to $S_{1'2'}$ and $S_{12'}$ are to be added. By introducing instead of r its value in figures, we get

$$\left. \begin{array}{l} 0{\cdot}894 \, S_{0'1} + S_{11'} = P, \\ 0{\cdot}486 \, S_{0'1'} - S_{11'} = 0, \end{array} \right\} \quad \cdot \quad \cdot \quad \cdot \quad (7)$$

S_{01} will disappear, its value being $= 0$. By these two equations between three quantities, $S_{0'1}$, $S_{0'1'}$, and $S_{11'}$, we are not able to ascertain any of these unknown values, until we introduce the displacings of the connection-points. The figure being symmetrical, the horizontal displacings will disappear, and there remain only the vertical ones, which we call v_1 for point (1) and $v_{1'}$ for point (1') and by which the strains must be expressed. Since $u = 0$ the former equation (5) gives

$$\rho = \frac{\beta \ (v_2 - v_1)}{r},$$

where v_2 and v_1 represent the displacings of the two extremities; and β is the piece of the vertical axis cut off by the lines which are drawn perpendicular from the extremities of the bar. (*See* Figs. 17 and 15.)

Fig. 17
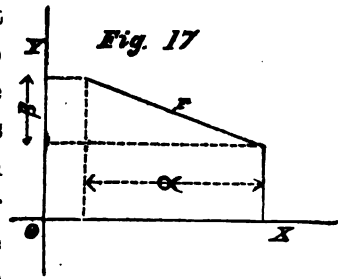
In the same way we find the prolongation of the bars :

$$\rho_{01} = 0,$$

$$\rho_{10'} = \frac{3\ a\ v_1}{r_{0'1}},$$

$$\rho_{0'1'} = \frac{\frac{1}{4}\ a}{r_{10'}} v_{1'},$$

$$S_{11'} = \frac{\frac{1}{4}\ a}{\frac{1}{4}\ a}\ (v_1 - v_{1'}) = v_1 - v_{1'}.$$

And for the strains we get :

$$S_{01} = 0,$$

$$S_{10'} = \frac{E\,q\,\rho_{10'}}{r_{10'}} = E\,q_{10'}\,\frac{\frac{1}{2}\,a\,v_1}{(r_{10'})^2},$$

$$S_{0'1'} = E\,q_{0'1'}\,\frac{\frac{1}{4}\,a\,v_{1'}}{(r_{0'1'})^2},$$

$$S_{11'} = E\,q_{11'}\,\frac{v_1 - v_{1'}}{r_{11'}}.$$

$$(8)$$

or by introducing the value of r:

$$S_{01} = 0$$

$$S_{0'1'} = E\,q_{0'1'}\,0\cdot235'\,\frac{v_{1'}}{a},$$

$$S_{0'1'} = E\,q_{0'1}\,0\cdot400\frac{v_1}{a},$$

$$S_{11'} = E\,q_{11'}\,4\frac{v_1 - v_{1'}}{a},$$

$$(9)$$

This introduced into the equations (7) gives

$$0\cdot3576\,E q_{0'1}\frac{v_1}{a} + 4\,E q_{11'}\frac{v_1 - v_{1'}}{a} = P,$$

$$0\cdot1142\,E q_{0'1'}\frac{v_{1'}}{a} - E q_{11'}\frac{v_1 - v_{1'}}{a} = 0,$$

or properly arranged

$$(0\cdot3576\,q_{0'1} + 4\,q_{11'})\,v_1 - 4q_{11'}v_{1'} = \frac{P}{E}a,$$

$$-4\,q_{11'}\,v_1 + (0\cdot1142\,q_{0'1'} + 4q_{11})v_{11} = 0.$$

$$(10)$$

By this we find v_1 and $v_{1'}$ as soon as we put a certain value for the sections; for instance, all equal $= q$, and we find

$$4\cdot3576\,v_1 - 4v_{1'} = \frac{P}{qE}a,$$

$$-4\cdot000\,v_1 + 4\cdot1142\,v_{1'} = 0,$$

$$(11)$$

which gives

$$\left.\begin{array}{l} v_1 = 2\cdot137\,\dfrac{P}{qE}a, \\[2mm] v_{1'} = 2\cdot080\,\dfrac{P}{qE}a. \end{array}\right\} (12)$$

These displacings are therefore proportional to the load P, to the length a, and inversely proportional to the modulus of elasticity, E.

The figure v_1 gives directly the deflection at the load, P.

As now the strains were expressed above by these displacings, we find by (9) and (12)

$$\begin{array}{l} S_{0'1} = 0\cdot8548\ P, \\ S_{0'1'} = 0\cdot4888\ P, \\ S_{11'} = 0\cdot2280\ P. \end{array}$$

This shows that the strains in the single bars are very different, and that we waste a great deal of material by making all bars equally strong, as previously supposed. To save, therefore, possibly, much material, we must take either the sections or the number of the bars different, so that the strain in all bars may be the same. The determination of the sections is the principal object of our calculation, and therefore we will look to this part more particularly, and will again use the above example, because the same rule is valid for all similar structures.

DETERMINATION OF THE SECTIONS.

To determine the sections of the bars in our structures, so that all bars have one *equal* strain per unit,

we are obliged to introduce into the calculation a
preliminary value for the sections. For example, to
simplify the calculations, we suppose the sections of
the bars all equal. Thus the calculation gives us cer-
tain strains, as shown above, and according to these
strains we fix the sections, so that one ☐ in. section
is exposed in each bar to an equal strain.

On this account we must know the load P. Let P
be equal 10 tons, and the whole structure be composed
of iron, loaded with 5 tons per ☐ in., the above strains
will be

$$S_{0,1} = 4\cdot89 \text{ tons,}$$
$$S_{0,1} = 8\cdot55 \text{ tons,}$$
$$S_{1,1'} = 2\cdot28 \text{ tons.}$$

This divided by 5 tons, gives the sections in square
inches :

$$q_{0,1'} = 1\cdot00 \ \square \ \text{in.}$$
$$q_{0,1} = 1\cdot71 \ \square \ \text{in.}$$
$$q_{1,1} = 0\cdot45 \ \square \ \text{in.}$$

By introducing these sections into the calculation, we
get different values for the strains, and consequently
we cannot call those the exact ones.

Looking back to equations (10) we introduce the
value of q, and find

$$2\cdot451 \, v_1 - 1\cdot84 \, v_{1'} = \frac{P}{E} \, a,$$
$$- 1\cdot84 \, v_1 + 1\cdot954 \, v_{1'} = 0 \; ;$$

and this gives

$$v_1 = 1\cdot393 \, \frac{P}{E} \, a = 13\cdot93 \, \frac{a}{E},$$
$$v_{1'} = 1\cdot312 \, \frac{P}{E} \, a = 13\cdot12 \, \frac{a}{E},$$

and the strains are now, according to (9),

$$S_{0'1'} = 3\cdot083 \text{ tons,}$$
$$S_{0,1} = 9\cdot528 \text{ tons,}$$
$$S_{11'} = 1\cdot490 \text{ tons.}$$

For equal sections, and for the same load (10 tons), the strains were before :

$$S_{0'1'} = 4\cdot89 \text{ tons,}$$
$$S_{0'1} = 8\cdot55 \text{ tons,}$$
$$S_{11'} = 2\cdot28 \text{ tons.}$$

We see the sections are very different, but also far more economical ; and still we cannot say these are quite exact. To make the strain in all sections $= 5$ tons per $\square$ in., the sections would be :

$$q_{0'1'} = 0\cdot62 \ \square \ \text{in.,}$$
$$q_{01'} = 1\cdot91 \ \square \ \text{in.,}$$
$$q_{11'} = 0\cdot30 \ \square \ \text{in.}$$

For these sections our strains would again be different, but not so much as before. By continuing this process, and by correcting the sections accordingly, we finally get the sections so, that in *every part* of the structure the strain will be exactly 5 tons per $\square$ in. if the point (1) is loaded with 10 tons.

In conclusion, we had to notice the dead weight of the bars, but this would cause, in comparison with the load of 10 tons, very little difference. In large structures, however, for example, girders for extensive bridges, at which we shall look afterwards, the weight

of the bar is surely of more importance, and must be taken to account in the calculation.

In the same way as shown above, all sections of these structures are calculated and fixed. The trouble caused by altering the sections different times is greatly simplified by choosing the sections at the first time in a suitable way, which is, after some experience, not at all difficult.

Thus I have also justified my former assertion, that these structures, even although they have two superfluous bars, can be calculated so that the strain per unit is the same throughout, and therefore these structures will be indeed structures of *equal strength* of material *in all their parts*—such as have not hitherto been produced, at least with the same perfection.

CALCULATION OF THE STRAINS WHICH ARE CAUSED BY CHANGE OF TEMPERATURE.

Exactly in the same way as we have previously determined the strains by aid of the displacings, we can also ascertain the strains caused by the change of temperature. As this determination is of the highest importance for bridges, I shall now explain its principles, especially as here an additional example in illustration of the previous theory is given.

The prolongation of a rod by heat is proportional to the increase of temperature and to the length of the rod. If, therefore, t represents the increase of temperature in degrees, r the length of the rod, and δ the co-efficient of expansion by heat, the prolongation of the rod will be

$$= \delta \, t \, r$$

The co-efficient δ has for different materials different values ; for iron, 100 δ is equal 0·001182 ; for steel not hardened

$$100\ \delta = 0·001079,$$

where 100 δ represents the prolongation per unit caused by an increase of temperature of 100° Celsius. By an increase of temperature of 30° a rod of 10 ft. length will thus be lengthened :

$$\frac{30 . 0·001182 . 10}{100}\ \text{ft.}$$
$$= 0·003546\ \text{ft.} = 0·0425\ \text{in.}$$

The prolongations produced by heat prove, therefore, to be very trifling.   However, they can produce a considerable strain.

If a rod be heated by $t°$, and prevented by exterior forces from expanding, a compressive strain will be produced in the rod, equal to the strain which is required to reduce to its original length a bar extended by $t°$.   This prolongation is however

$$\rho^\wedge = \delta\ r\ t,$$

and the corresponding strain will be

$$\hat{S} = -\frac{E\ q\ \rho^\wedge}{r} ;$$

or by introducing the value of $\rho^\wedge$

$$\hat{S} = -E\ q\ \delta\ t,$$

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and for the strain in the section of a unit

$$\frac{\hat{S}}{q} = E\,\delta\,t,$$

where E is the modulus of elasticity.

An iron rod of 10 ft. length, prevented from expanding, and heated by 30°, will *try* to expand with a force

$$-\frac{\hat{S}}{q} = +\,12800 \cdot \frac{0\text{·}001182}{100} \cdot 30 \text{ tons.}$$

$$= 4\text{·}538 \text{ tons per } \square \text{ in.,}$$

even although its expansion, were it unopposed, would only amount to

$$0\text{·}0425 \text{ in.}$$

If instead of an increase there is a decrease in the temperature, the rod will be shortened, and if it likewise is prevented from doing so a tensile strain will be produced. The formulæ will be the same for decrease as for increase, and will only change their signs. I shall, therefore, only consider the effect of increase in the temperature : that for decrease in temperature can easily be obtained by introducing $-\,t$ instead of $+\,t$.

If the force which tries to prevent the prolongation of a rod, is not quite able to do so, the heat will also prolong the bar a little, by which the expanding force will be diminished. Thus it will arrive at a point where, between the exterior force and the expanding heat, equilibrium is produced.

If the heat with this resistance can thus expand

the rod by the prolongation ρ', we may say that this force has shortened the bar by $\rho^\wedge - \rho'$ after the heat had expanded it previously by $\rho^\wedge$. The force by which the rod still tries to extend corresponds with the shortening $\rho^\wedge - \rho'$. If we call this force S, and we

$$\rho' - \rho^\wedge = \rho \qquad (13)$$

we get

$$S = \frac{\rho \cdot E\,q}{r} = \frac{(\rho' - \rho^\wedge)\,E\,q}{r}$$

and according to the previous

$$S = E\,q\ \left(\frac{\rho'}{r} - \delta\,t\right)$$

as :

$$\frac{\rho^\wedge}{r} = \delta\,t\,.$$

Putting now

$$E\,q\,\frac{\rho'}{r} = S' \text{ and } \frac{E\,q\,\rho^\wedge}{r} = \hat{S}$$

then also is :

$$S = S' - \hat{S}. \qquad (14)$$

In this S represents the strain corresponding to the whole prolongation produced by heat ; S' a strain corresponding to the actual prolongation, whilst $\hat{S}$ signifies the strain acting in the rod.

The equation (13) will not alter its value by introducing a tensile instead of a compressive force ; only the prolongation ρ' alters its sign, and hence we get

$$\rho = \rho' + \rho^\wedge$$

and

$$S = -\,(S' + \hat{S}).$$

If we look now at the strains caused by heat in a suspended or trussed beam, or any other similar structure, which can not expand or contract freely, we may use the same equations as before, which signify that the strains in the connection-points are in equilibrium.   These strains were formerly called S, for which we must now substitute $S - \hat{S}$. We know $\hat{S}$, as it depends only on the increase of temperature, whilst S will be expressed as above by the displacings of the connection-points. Thus we get instead of the equations for the different S those of the displacings, and exactly so many as necessary. We can therefore examine the action of heat in the same manner and with the same exactness as we did previously the strains.*

Example.

As an example, we shall take the same structure as that for which we previously calculated the strains, and will consider this example in all points, as it is the simplest case of such a suspended bridge as is afterwards described, the calculation of which can be executed in the same way.

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* In a similar way we may find the strains which arise in a structure if a superfluous rod is too short or too long (as shown in page 12). We suppose the rod is extended or contracted to the length which it ought to have without any strain within. This causes a strain which acts on the connection-point of the bar. The conditions of equilibrium for the connection-point give them the strains which arise in the various bars, similar to that produced by the influence of heat, with the exception that here originally only a single bar strives to alter its length, whilst by heat a force originally acts in each bar.

By omitting in the above equations (7) the load P, we get:

$$0\cdot894\ S_{0\prime1} + S_{11\prime} = 0,$$
$$0\cdot486\ S_{0\prime1\prime} - S_{11\prime} = 0.$$

S being equal $S' - \hat{S}$ where $\hat{S}$ represents the strain produced when the rod cannot expand at all.

Now $\qquad\qquad \hat{S} = E q\ \delta\ t,$

where δ indicates the co-efficient of expansion in the material, and t the increase of heat in degrees Celsius. For iron

$$\delta = 0\cdot00001182\ ;$$

and by taking $t = 30°$ Celsius,

$$\frac{\hat{S}}{q} = 4\cdot574\ \text{tons};$$

so that the equations will now become,

$$0\cdot894\ S'_{0\prime1} + S'_{11\prime} = 0\cdot894\ \hat{S}_{0\prime1} + \hat{S}_{11\prime},$$
$$0\cdot486\ S'_{0\prime1} - S'_{11\prime} = 0\cdot486\ \hat{S}_{0\prime1} - \hat{S}_{11\prime}.$$

To introduce the value of $\hat{S}$ we must know the bar sections, and for greater simplicity I shall put them all $= q$. Thus all $\hat{S}$ will be $= 4\cdot574\ q$ tons, and the equations become,

$$0\cdot894\ S'_{0\prime1} + S'_{11\prime} = \qquad 8\cdot663\ q\ \text{tons},$$
$$0\cdot486\ S'_{0\prime1} - S'_{11\prime} = -\ 2\cdot351\ q\ \text{tons},$$

where the S' must be expressed by the displacings. As the S' are the strains, which correspond to the

actual displacings, they depend upon these in the same
way as previously S did.   See formula (8).   By this
our equations change into

$$4\cdot3576\ \dot{v}_1 - 4\ \dot{v}_{1\prime} = 8\cdot663\ \frac{E}{a}\ \text{tons},$$

$$- 4\ \dot{v}_1 + 4\cdot1142\ \dot{v}_{1\prime} = - 2\cdot351\ \frac{a}{E}\ \text{tons},$$

which correspond with the previous (11).

They give the values

$$v_1 = 13.609\ a,\ \text{and}\ v_{1\prime} = 12\cdot660\ a.$$

and the equations (8) will give the values of

$$S'_{0\wedge1} = 5\cdot444\ q\ \text{tons},$$
$$S'_{0\wedge1\prime} = 2\cdot975\ q\ \text{tons},$$
$$S'_{11\prime} = 3\cdot796\ q\ \text{tons}.$$

Because $$S = S' - \hat{S}$$

the strains actually produced by heat become

$$S_{0\wedge1} = + 0\cdot870\ q\ \text{tons},$$
$$S_{0\wedge1\prime} = - 1\cdot599\ q\ \text{tons},$$
$$S_{11\prime} = - 0\cdot778\ q\ \text{tons}.$$

These figures give us the strains per ☐ in.   Under
a load of five tons per ☐ in. the greatest amounts to
1·6 tons.

We must therefore in this, as also in all similar
structures, where a free expansion is prevented, increase
the sections according to the strains.

These strains are caused by an increase in temper-
ature of 30° Celsius; for an equal *decrease*, we get

SKELETON STRUCTURES. equal but reverse values ; *i. e.* instead of tension we get compression, &c.

These calculations can easily be transferred to a temperature of any degree. Thus for 1° the above results must be divided by 30, and thus a decrease in temperature of 10° gives

$$S_{1'0} = -\ 0\text{·}290\ q \text{ tons},$$
$$S_{0'1'} = +\ 0\text{·}533\ q \text{ tons},$$
$$S_{11'} = +\ 0\text{·}259\ q \text{ tons}.$$

BRIDGES.

Of all constructions for bearing or support that come before the engineer, those for the support of bridges are the largest and most important. These are, therefore, most suitable to illustrate the value of the Skeleton Structures, which in the future will probably be much used in the construction of bridges. I will now, therefore, enter into this part of Engineering more minutely.

In designing bridges and similar structures the first question is, Which construction is absolutely the best for the intended object? This the engineer can never answer. His task is, therefore, to compare all the various known constructions with each other, and to balance their respective advantages and disadvantages. By this, though he can never discover *the best possible*, yet he may arrive at that which of the known ones is the *best for his object;* but he can never thereby affirm that the one he has fixed on may not be replaced by a better.

Here, therefore, where it is my chief object to

explain a new system for such structures, I will try to compare the new construction with the best existing one. If by this comparison we find that this construction can, with equal strength, be executed more cheaply, or that with equal amount of material, it gives greater strength, then must this principle be acknowledged as *theoretically* the better ; and if no difficulties occur in the construction, and if the strength of the girder is not affected by tear and wear, it must also be acknowledged as *practically* the better.

In the first part I have already directed attention to the advantages derived from the connection made *entirely* by bolts, and proved that the material is best applied in this way ; and that bridges so constructed must be lighter than any other. I have there proved how the sectional areas of the structure are determined, and will now compare the weight of girders thus found with that of other girders. This is the only point we have still to examine, as the connection by bolts, on which the strength and permanence of these structures depend, has already been considered in detail.

Two points of comparison with other bridges present themselves : firstly, the weight of the girder in proportion to the greatest load which it can carry, upon which depends principally the cost of the structure ; and, secondly, the deflection in proportion to the span. This *deflection* is the only measure we have of the rigidity in a bridge, until we solve the problem to ascertain the oscillation of a bridge whilst a train is passing, or until a bridge has been built

and tested for this purpose. Such trials, in which the oscillations and deflection are exactly observed, are, I am convinced, made only in very few bridges ; as in works otherwise so rich in information about bridges, we find seldom data of the deflection for various loads.

Both weight and deflection depend to a very great extent on the shape of the girder, and thus the shape is of the greatest importance. It is difficult to say which scheme is the best, as this will greatly depend on the purpose of the bridge. If greater lightness is required, so must it be procured by loss of rigidity and increase of deflection ; whilst, on the contrary, the deflection can be lessened by employing more material.

The bridges which can be built as Skeleton Structures are of the most various kinds. We find in the annexed plates examples of beam-bridges, which cause *no* horizontal pressure on the abutments (Figs. 7 and 8, Plate III.) ; further, of suspended and trussed bridges, remarkably perfect in their construction (Plate I. and II.). All these bridges have top and bottom members, which are so braced together that the whole girder forms a rigid mass (as considered, page 10, &c.).

I shall enter now upon a closer examination of the suspended and trussed bridges ; and principally the simplest of these (*see* Fig. 1, Plate I.), where it will be observed that the trussed girder is the reverse of the suspended.

SUSPENDED BEAMS.

The suspended beam (Fig. 1, Plate 1) is the clearest representation of all peculiarities of this mode of con-

struction. Such a beam fixed at two points in each pier forms a perfect rigid body. Wherever we might cut off the beam, the piece between the pier and the section will keep its position unaltered. In cutting the bridge for example in the centre, each half will form an arm of a girder, as is shown in the swing bridge (Fig. 3, Plate I.). Similar girders are obtained by cutting the bridge anywhere, for it is easily seen that nowhere is a rod wanting, and that therefore the structure is perfectly rigid ; neither is there any superfluous rod as soon as the section is made. In considering, however, the whole structure, we find in reality two rods too many, as both ends, altogether four points, are fixed. Through this, however, can in no case any special strain arise. If, for example, in the lower member, a bar should be a little too long, the strain produced would be distributed over the whole length of the member, and if this consist of 10 bars, each bar would be exposed to one-tenth of the strain. If, thus, a rod be too long by 1000th of its length, the strain would become very little, and can be neglected. Besides, the mode of constructing such bridges would diminish the strain.

The erection of the bridge will proceed from each pier, by which means the bridge always supports itself. Arrived at the centre, the closing bars may be bored on the spot. When erecting a bridge in this manner there would be no especial need of scaffolding or other support, as the roadway of the bridge may at once serve as such. We need not make any further observations to show the great advantages of this mode of erection, particularly for

bridges of great length and across deep chasms, where
an intermediate temporary scaffolding causes the
greatest difficulties.

Some bars, however, may be subjected to greater
strains before the bridge is connected in the centre
than afterwards. A simple calculation will show if
the bridge, before shutting, is able to carry its own
weight without injury to one of those bars, and if in
large spans this should not be the case, it will be easy
to procure temporary chains to support the weight.

The sectional areas of each separate bar are deter-
mined according to the explained theory by calcula-
tions as follows : Firstly, we assume that the load on
the bridge is equally distributed. This load acts on
the connection-points, and only there, as the cross
girders carrying the platform rest upon the bolts.
We have, therefore, on each connection-point of the
lower member a load P acting, in which the weight
of the platform, as well as casual loads, is included.
The latter is supposed equal to the greatest load
which the bridge shall carry.

In the following it is assumed that this load, in-
cluding the weight of the platform, is for a single line
of rail = 2 tons per running foot. In proportion to
this uniform load the strains in all bars, and by this
their sections, will be determined, as shown page 46.
According to these sections we calculate the strains
for *eccentric loads*, and correct those sections which are
thus found to be too small. These sections will be
able to resist any load, but would need a final correc-
tion to resist the influence of temperature. As the
whole body of a girder forms a strong beam, which is

fixed at each end, the bridge, by change of tempera-
ture, cannot expand or contract freely ; and all the
strains caused by these changes must be resisted
within the structure.   How these strains are deter-
mined is shown page 48.   The magnitude of these
strains depends chiefly on the climate of the country
where the bridge is to be erected ; and is, of course,
very different.   If the bridge is erected during the
middle temperature of the year, then we have to take
into our calculation the highest and lowest tempera-
ture of the country.   I have assumed in the former
examples that the bridge is erected at a middle tem-
perature, and then taken 30° Centigrade, = 54° Fah-
renheit, above and below it; so that the bridges have
been submitted to all fluctuations of temperature
within 60° Centigrade = 108° Fahrenheit.

To give an idea how these strains thus found by
calculation are distributed in these suspended bridges,
I have prepared some diagrams (shown in Plate IV.),
in which the amount of strain is signified by the
thickness of the respective lines.   At the same time
I have chosen for compression and tension different
colors, thus, that all ties are shown black, and all
struts are shown red.   The bars represented by the
thickest of the red lines have therefore to resist
the greatest compression, and the bars of the thickest
black lines likewise the greatest tension, according to
the circumstances mentioned afterwards.

Fig. 15 gives, thus illustrated, the strains in a

bridge girder of 200 ft. span uniformly loaded. The inserted numbers give the amount of strain in tons of each bar ; if the girder is thus loaded, that dead weight and rolling load amount to one ton per running foot. By taking steel of 10 tons admissible strain per $\square$ in., the dead weight of one girder will be $\frac{1}{10}$ ton, therefore the rolling load, with platform, will amount to $\frac{9}{10}$ ton per running foot.

The dotted lines are intended to illustrate the deflection caused by this load ; they are drawn according to the displacings calculated for each connection-point. As these displacings, however, are so trifling that they could not be shown in the same scale as the girder, they are drawn five times enlarged—i.e. the real deflection is $\frac{1}{5}$ of that one shown.

Fig. 16 gives for the same bridge and the same material the strains which occur when the temperature rises by 30° Centigrade ($=$ 54° Fahrenheit). By this, all strains which are caused by dead weight and load are not taken into account; the diagram shows, therefore, those strains which are produced by the heat only. These are to be added to the strains caused by the load, which combination is shown in Fig. 17. This diagram contains, therefore, the strains which occur by a uniform load, and by an increase in temperature of 30° Centigrade.

By determining the latter strains, I have considered the most unfavorable way of connection—viz., that the ends of the girder are fixed absolutely in the piers. In practice, these strains will be considerably less.

In Fig. 16 the change of form is also indicated by dotted lines ; but as these are considerably smaller

than those in Fig. 15 they are 50 times enlarged.
To know this deflection is but of little interest, only
so far as it would be advisable to give the roadway of
the bridge a raise in the centre, at least equal to the
greatest possible deflection, so that the girder can
never deflect below the horizontal line.

As now the deflection by heat is to be distinguished
from that one caused by the load, I have not shown
any deflection in Fig. 17. The deflection by heat is
the same as shown in Fig. 16 (*i.e.* $\frac{1}{10}$ part of it), and
that one momentary produced by the load is the same
as in Fig. 15 (*i.e.* $\frac{1}{4}$ part of it). The latter, as before
mentioned, can be considered as a measure for the
rigidity of the structure; and this rigidity does not
suffer by the action and deflection caused by heat.

In regard to Fig. 16, it might be mentioned that a
decrease in temperature of 30° Centigrade does not
alter the *amount* of strain, but compression will be-
come tension, tension become compression; and in-
stead of a deflection, we get a convex curve. Fig. 16
gives also, therefore, an illustration of the state at 30°
Centigrade decrease in temperature—only the black
lines signify in this case compression, and the red ones
tension.

After the calculations were completed in the above
way, all sections were so determined that each square
inch at the highest admissible strain (*i.e.* the strain
caused by the joint action of the most unfavorable
load and most unfavorable influence of temperature),
will carry exactly so much as its material will allow.

In this way, we find the smallest sections with which the bars can resist most safely all the strains to which they are exposed. In practice, however, it would not be advisable to take each bar of a section exactly fitted to its strain, but to introduce different numbers of bars of equal section ; and such a section ought to be taken that in all parts there is rather more than less of the required material.

In this manner I have calculated several bridges, and give here the data which were found for a bridge of 100 ft. span (Fig. 1, Plate I.), where various materials, from ordinary iron to the best steel, are supposed to be used in the structure. By composing a table of weights of different materials, we will see at once the advantage of a superior sort of metal.

The value of material for such structures depends chiefly on the greatest strain that one ☐ in. of it is able to bear. We called it above the greatest admissible strain, and will indicate it by T, which always relates to one ☐ in. Besides this quantity, the "modulus of elasticity" must be taken into consideration in calculating the deflection, but has no influence upon the weight of a bridge.

I will therefore classify the material by its highest admissible strain, and indicate it as that for which T has the value of 5 tons, 6 tons, &c. I will also take into my calculations all the values of T from 5 to 12 tons, the former being a safe load for ordinary iron, whilst the latter is the same for unhardened steel. These numbers include the best qualities of iron and the newer sorts of half-steel and steel, which have lately been introduced in the trade.

The greatest admissible compression per ☐ in. is, as usual, taken a little less than the tension, namely, = $\frac{5}{6}$ T, though I am of opinion that even wrought iron can resist a compressive equally as well as a tensile force ; provided that all bending out or twisting of the material is prevented. For half-steel and steel both strains can nearly be taken the same. As it is, however, difficult to prevent totally bending out, I have, as said before, taken everywhere the sections so that the greatest compression becomes nowhere more than $\frac{5}{6}$ of the greatest tension.

I give first the weights of a bridge of 100 ft. span in the following table :

TABLE I.

T in Tons.	Weight of One Girder in Tons.	Weight of the whole Bridge in Tons.
5	10·7	41
6	8·8	38
7	7·4	35
8	6·4	33
9	5·7	31
10	5·1	30
11	4·6	29
12	4·2	28

The first column of this table gives the greatest admissible strains which the materials can resist, or, if you like, the highest strain caused in a bar by the most unfavorable load and influence of temperature.

The second column gives the weight of *one* girder of 100 ft. span for the various materials, where it is

assumed that such a beam can carry, besides its own weight, 1 ton per running foot, unless the corresponding number of the first column be overstepped. Consequently, a bridge having two girders can carry, together with the platform, 2 tons per running foot; which is a heavy load for a single line railway bridge.

From this table we can very easily find the weight of one girder required to carry any given load, as we have only to multiply the numbers of the table with the load per running foot. As the general load for railway bridges of middle spans is $1\frac{1}{4}$ tons per running foot, each girder would carry only $\frac{5}{8}$ tons of rolling load. To this must be added the weight of the platform, which I take $= \frac{1}{5}$ ton per running foot,[*] and thus both girders have, besides their own weight, $1\cdot25 + 0\cdot2 = 1\cdot45$ tons, and each girder $0\cdot725$ tons. If we take, then, $0\cdot73$ tons per running foot, we would receive the following table, which gives also the weights of a 100 ft. bridge able to carry, besides its own weight, $1\frac{1}{4}$ tons per running foot.

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[*] This number, generally assumed for wrought-iron plate girders, must not be supposed to be too low, as lighter girders have been made.

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TABLE II.

Weights of One Girder of 100 Feet Span, when Two Girders carry, besides their own Weight and that of the Platform, 1¼ Tons per Running Foot.

T in Tons.	Weight of One Girder in Tons.
5	7·8
6	6·4
7	5·4
8	4·7
9	4·2
10	3·7
11	3·4
12	3·1

If one girder weighs 7·8 tons, two girders would weigh 15·62 ; the platform $=$ 20 tons ; and, therefore, the total weight of the bridge would be $=$ 35·6 tons, or $=$ 0·356 tons per running foot ; and this carries besides 1·25 tons ; in all, therefore, 1·606 tons per running foot, *i.e.* $\frac{1·606}{0·356} = 4·5$ times its own weight. If we make, however, bridges in steel of 10 tons admissible strain, the weight of the whole bridge will be only 27·4 tons, which includes 20 tons for the platform.

For lattice and plate girders, we assume that the average weight of two girders for single line is in lbs. nine times the span in feet. For a 100 ft. the weight of two girders is thus 900 lbs., or for *one* girder 450

lbs., or 0·201 tons per running foot, making 20 tons for the whole girder.

As now, however, lately, such lattice-girders have been built sometimes lighter, the average weight of a 100 ft. bridge would scarcely be taken below 0·15 tons per running foot, which makes the weight of one girder = 15 tons, being about twice the weight of a skeleton girder. If the platform weighs 20 tons, and each girder 15 tons, the whole bridge will thus be = 50 tons, whilst ours is only 36 tons.

I add here the calculated deflection of a loaded girder of 100 ft. span. A 100 ft. bridge for single line, in iron of 5 tons admissible strain, will deflect under a load of $1\frac{1}{4}$ tons only 0·63 inches. This is very nearly $\frac{1}{1800}$ of the span, whilst most railway bridges have a greater deflection under such a heavy load. If I take half the above load, or $\frac{5}{8}$ tons per running foot, say the weight of a goods train, the deflection would also be only the half, i.e. $\frac{1}{3}$ inch, and for lighter trains proportionally less.

As now the deflection and also the strains as above explained are found with great exactness, so would these numbers be the strongest proof that these bridges, in spite of their great lightness, possess sufficient interior rigidity to carry both speedy and heavy trains with the greatest safety.

In steel bridges the deflection is of course greater, as is easily observed from the previous formula (2), for the extension of a bar. According to these the prolongation is

$$\rho = \frac{r\,S}{E\,q}.$$

This quantity increases therefore in proportion to the strain per ☐ in., and is reversely proportional to the modulus of elasticity, E. Now the modulus of elasticity for iron and steel varies between 28 and 30 millions of lbs., where hardened steel is excluded. I have taken it always = 12,900 tons, or about 29 millions of lbs. The prolongation depends, therefore, only on the strain per ☐ in., $\dfrac{S}{q}$, and this is always greater in the case of steel.

A 100 ft. bridge made of steel, with 10 tons admissible strain per ☐ in., suffers, therefore, by a load of 1¼ tons per running foot, a deflection of 2 × 0.63 = 1·26 or 1¼ in. ; *i.e.* of about $\frac{1}{1000}$ of the span.

These bridges would for ordinary trains be sufficiently rigid ; at any rate the deflection of steel bridges constructed otherwise will not be less, if they are of the same weight.

If we, however, would build a steel bridge of less deflection, we have only to increase the section of the bars. If the girder be made heavier by half of its weight, the deflection will be one-third less. Therefore, in each structure, the special object must decide the nature and amount of material.

For the other materials given above we get the following deflections :

TABLE III.

Deflections of a 100 *Feet Bridge of the same Weight as that in* Table II., *and loaded with* 1¼ *Tons per running Foot.*

T in Tons.	Deflection through Loading.		
5 .	0·62 inches	= 0·00050 of span.
6	0·76 "	= 0·00063 "
7	0·90 "	= 0·00075 "
8 .	1·04 "	= 0·00087 "
9	1·18 "	= 0·00098 "
10	1·31 "	= 0·00109 "
11	1·45 "	= 0·00120 "
12	1·59 "	= 0·00130 "

The deflection caused by the above load is, therefore, only in materials of 10, 11, and 12 tons admissible strain greater than $\frac{1}{1000}$ of the span.

To these data I add a table for the deflection of free-supported girders, Fig. 18, on which the load is also equally distributed. For such a beam we may take the simplest —a plate girder of equal section throughout. In this case the deflection

Fig. 18.
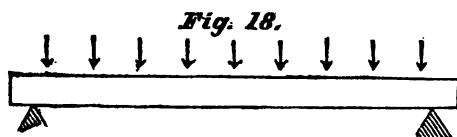
$$\delta = \frac{5}{8} \frac{P}{EJ} \frac{L^3}{48},$$

where P represents the load per running foot, including the dead weight, L the span, and J the momentum of inertia of the section " for the horizontal axis."

If otherwise in a beam T is the highest strain per ▢ inch, the load P can be taken as large, as it follows from the formula :

$$16 \frac{J}{h} T = P L,$$

in which it is supposed that the centre of gravity in the cross section of the beam is as half its height, h. From this we see

$$\frac{L}{J} = 16 \frac{T}{h} \frac{1}{P}.$$

This being introduced into the above equation makes

$$\delta = \frac{5}{24} \frac{P}{E} \frac{L^2}{16} \frac{16}{h} \frac{T}{P} \frac{1}{P},$$

or, $$\delta = \frac{5}{24} \frac{P}{E} \frac{L}{H} \cdot L \, ;^*$$

where, however, δ now represents the deflection of such a beam, when so loaded that the greatest admissible strain is reached.

If again we put $E = 12,900$ tons ; for T the respective values 5, 6, ... 12 tons; and lastly, as on an average is the case in bridge-girders $h = \frac{1}{10} L$, or $\frac{L}{h} = 10$, we get the following table :

---

* Laissle & Schübler, " Bau der Brückenträger "—page 73.

TABLE IV.

T in Tons.	Deflection at 100′ Span in Inches.	Deflection in proportion to the Span.
5	0·96	0·00081 . L
6	1·19	0·00097 "
7	1·35	0·00123 "
8	1·54	0·00129 "
9	1·74	0·00145 "
10	1·93	0·00161 "
11	2·12	0·00177 "
12	2·33	0·00194 "

As in Table III., it has been here also supposed that the girder is so loaded that the greatest admissible strain T is reached. These tables are therefore very suitable for a comparison between the deflections, and this comparison results decidedly in favor of the new bridges. The load which a beam with such a deflection can carry depends, of course, also on the form of cross-section.

This 100-feet bridge I have examined thoroughly in order to explain the character of the whole system. Henceforward I can be more concise, as I now proceed to give the weights of girders for different and larger spans, and to compare these with some of the executed bridges.

I give firstly a few tables exactly like I. and II., only for different spans.

The weights in these tables are for girders of geometrical-similar form, so that the design, Fig. 1, Plate I., will obtain for all these girders, since they

may be drawn to different scales. The sections, and by these also the weights, are then from a 100-feet bridge determined by calculation, by the condition that the same maximum load per running foot and the greatest strain will be the same as in those.*

It will be seen that in these tables no regard is paid to the span at which these bridges may be built, because this depends chiefly on the greatest length of the bars or members which in practice may be adopted with success. I consider 700 feet to be the limit for this kind of bridge, if the division points are properly arranged. These divisions depend on the distance of the cross-girders, and whether these· are connected by longitudinal girders or not—as the cross-girders can only rest on the bolts of the connection-points.

For large spans, therefore, another design is necessary in which the length of the bars is in general less, and this is the case in Fig. 4, Plate II., where we have two curves.

The average weights of these bridges may be taken from the same tables as the others.

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* How these weights are found by those of the 100-feet bridge—*see* Appendix.

TABLE V.

*Containing the Weights of One Bridge Girder, which, be-
sides its own weight, can carry 1 Ton per running
Foot, including the platform.*

T	G_{100}	G_{200}	G_{300}	G_{400}	G_{500}	G_{600}	G_{700}	G_{800}
	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.	Tons.
5	10·7	48·0	122·4	252·8	467	828	1465	2726
6	8·8	38·4	95·4	190·4	337	562	907	1450
7	7·4	32·0	78·3	152·3	262	425	657	987
8	6·4	27·6	66·4	127·5	217	342	515	749
9	5·7	24·0	57·9	109·6	185	284	421	593
10	5·1	21·2	50·8	95.8	160	245	358	504
11	4·6	19·2	45·6	85·4	140	216	309	433
12	4·2	17·6	41·3	76·8	125	191	274	381
13	3·9	16·0	37·6	69·9	115	173	245	329

If we wish to find the weight of a girder for another
load we have to multiply the respective number in the
table by the load per running foot. Suppose, for
example, the problem is to find the weight of a girder
for a single-line railway bridge, loaded with 1¼ tons
per running foot, if the span be 300 feet, and the
admissible strain 8 tons per square inch,

The two girders carry   -   -   1·25 tons.
Platform per running foot -   -   0·2   "

Total load   -   -   -   1·45   "·

i. e. 0·73 tons per running foot for one beam, besides
its own weight. In a span of 300 feet, and under a

strain of 8 tons per ☐ inch, the table gives a weight of 66·4 tons, and this multiplied by 0·73 gives for the beam in question a weight of 48·57 tons. Two girders weigh, therefore, 97 tons, so that the whole bridge weighs, with the platform of 60 tons, 157 tons.

In this way is the sixth table arrived at.

TABLE VI.

Giving the Weights of One Beam when Two of them, besides their own weight and that of the Platform, can carry 1¼ Tons per running Foot.

T	100	200	300	400	500	600	700	800	900	1000
5	7·8	35·0	89·4	184·4	337	604	1070	1990	4393	21160
6	6·4	28·0	69·6	138·8	247	410	661	1058	1733	3030
7	5·4	23·4	57·3	110·8	191	310	480	721	1076	1628
8	4·7	20·2	48·3	93·6	159	250	375	546	780	1117
9	4·2	17·6	42·0	80·4	135	208	303	425	615	847
10	3·7	15·4	36·9	70·0	117	179	271	368	502	679
11	3·4	14·0	33·3	62·0	102	158	225	316	432	569
12	3·1	12·8	30·3	56·0	93	139	200	278	373	489
13	2·8	11·6	27·3	51·2	84	126	179	248	331	438

To this I still add a table giving the same weights as in the previous one, not, however, for the whole beam but for the running foot, with the purpose of gaining better means of comparison.

TABLE VII.

Containing Weights per running Foot of different Girders, when Two of them, besides their own weight and Platform, have still to carry 1¼ Tons per running Foot.

T	100	200	300	400	500	600	700	800	900
5	0·078	0·175	0·298	0·461	0·673	1·007	1·528	2·488	4·881
6	0·064	0·140	0·232	0·347	0·493	0·683	0·945	1·323	1·925
7	0·054	0·117	0·191	0·277	0·382	0·517	0·685	0·901	1·196
8	0·047	0·101	0·161	0·234	0·317	0·416	0·537	0·683	0·867
9	0·042	0·088	0·140	0·201	0·270	0·346	0·433	0·541	0·683
10	0·037	0·077	0·123	0·175	0·234	0·298	0·373	0·460	0·558
11	0·034	0·070	0·111	0·155	0·204	0·263	0·322	0·395	0·480
12	0·031	0·064	0·101	0·140	0·183	0·232	0·286	0·347	0·414
13	0·028	0·058	0·091	0·128	0·168	0·210	0·256	0·310	0·368

T	1000	1200	1400	1600	1800	2000	2200	2400	2600
5	21·160								
6	3·030	21·155							
7	1·628	3·534	21·203						
8	1·117	1·925	4·016	21·193					
9	0·847	1·323	2·212	4·461	21·429				
10	0·679	1·008	1·529	2·491	4·887	21·192			
11	0·569	0·814	1·175	1·728	2·777	5·294	21·188		
12	0·489	0·683	0·944	1·322	1·898	3·024	5·681	21·185	
13	0·438	0·588	0·792	1·072	1·477	2·117	3·280	6·047	21·183

These tables, especially the latter, give a clear view of the weights of girders. We see from them how the weight of the girder itself increases rapidly compared with the span.

Further, we see that for larger spans the bridges

of better material are much lighter than those of in-
ferior, which shows the advantage of better material,
especially in large structures.

To assist us in forming a better opinion of the
material, the following table will be found interesting.
It contains the spans for different materials when a
girder can carry nothing but its own weight :

TABLE VIII.

Admissible Strain.	Theoretical Limit of Span.
5 tons - - - - - - -	1035 feet,
6 " - - - - - - -	1241 "
7 " - - - - - - -	1448 "
8 " - - - - - - -	1655 "
9 " - - - - - - -	1862 "
10 " - - - - - - -	2069 "
12. " - - - - - - -	2276 "
12 " - - - - - - -	2483 "

At these spans a bridge will not be able to bear any
load without exceeding the greatest possible strain of
material.

In practice it will be advisable to keep far below
these figures, because the dead weight of the girders
increases rapidly as you approach these spans ; as
Table VII., for $T = 5$ has also shown, giving for 900
feet span 4·88 tons dead weight per running foot, and
21·16 tons for 1000 feet span ; and by going 35 feet
further the dead weight becomes infinite.

Although, therefore, on the one hand, a bridge of
1000 feet span can scarcely be built of wrought iron,
on the other it cannot be denied that according to

this plan larger spans, excluding common road sus-
pension bridges, will be possible. But by using
steel we are able to exceed the above span. From
the use of this metal less material is required, and
this saving increases remarkably in crossing large
distances.

It is still a question, which is the most suitable
arrangement : short spans and many pillars, or long
spans and few pillars? It is difficult to speak de-
cidedly on this point, because there are many circum-
stances to be taken into consideration. One thing
is clear : that the most suitable span will increase
remarkably if we are able to build higher bridges,
i. e. having less dead weight, and this will be proved
in every respect by adopting this skeleton construc-
tion.

In conclusion, I have to compare the bridges of
skeleton construction with the structures already
executed, more particularly as I used before only aver-
age values of plate and lattice girders.

By aid of the above tables I shall calculate the
weight of a bridge of the same span as that of the
one already erected, with which it is to be compared.

The largest span which has hitherto been adopted
with success for railway traffic we find in the *Britannia
Bridge*, which, by the novelty of its construction and
its audacity in erection, caused not only general admi-
ration at the time, but will always be considered a re-
markable epoch in the history of iron structures.

This bridge has a span of 460 feet, which is crossed
by a tube of 1400 tons weight for single line, making
3·043 tons per running foot.

One skeleton girder, of 460 feet span single line, made of wrought iron, loaded with 4 tons per ▢ inch, and which carries besides the roadway (= 0·2 tons per running foot), 1¼ tons per running foot, weighs 0·584 tons.

> Which makes for two girders . 1·168 tons
> Roadway   .   .   .   .   0·2   "
>
> Total weight per running foot . 1·368   "

Which shows that a Skeleton Bridge would weigh 1·675 tons less per running foot than the Britannia Bridge.

In connection with this we have to look at the following points :

Firstly, the weight of our roadway is, according to the present mode of construction, rather light, as a bridge at such a span must be provided with suitable horizontal bracing to resist the side action of the wind. These bracings might be common tie rods, or arranged as shown in Fig. 6, Plate II. The weight of such an arrangement is, however, not taken into account in the above calculation.

Secondly, we must consider that this bridge is able to carry 1¼ tons per running foot safely, being nearly equal its dead weight. The Britannia Bridge carries, besides its own weight, about 1 ton per running foot.

We further have neglected, in our Skeleton Bridge, the weight of our pillars and abutment-bars ; but we have also neglected the cast-iron end frames in the Britannia Bridge, which will pretty nearly balance each other. Because by taking the whole length of 4 spans, two of which are only 274 feet, we find a

total weight of 5285 tons in a length of 1524 feet, which is 3·468 tons per running foot, being nearly ½ a ton more than in the above calculation. By taking everything into consideration we shall not be far wrong in stating that a Skeleton Bridge of the same span as the Britannia Bridge will want about half the material. This is especially remarkable, as the bottom of the tube in this bridge, representing the roadway, belongs chiefly to the carrying part of the structure, which is not the case in our bridge, where the roadway does not carry, but increases the load.

The deflection of a skeleton single-line bridge, being loaded with 1¼ tons per running foot, amounts, according to my calculation, to 2·85 inches, which will completely disappear after the rolling load has passed. By taking in such a length of bridge ½ ton per running foot for an ordinary goods train, the deflection will be 1·14 inches, whilst one tube of the Britannia Bridge, at a load of 248 tons (27 coal wagons) being about ½ ton per running foot, deflected 0·676 inches. According to this, the deflection at 1¼ ton load per running foot would be 1·69 inches. By making, however, our Skeleton Bridge double the weight, i.e. about equal to the weight of the Britannia Bridge, the deflection of our bridge will be 1·42 inches at 1¼ tons load, and 0·55 inches at ½ ton load per running foot, showing a smaller deflection at equal weights.

I will add here the span which would be crossed by a skeleton girder having the same weight as one tube of the Britannia Bridge of 460 feet length = 3·04 tons per running foot. Taking the weight of the road-

way, including horizontal bracing, = 0·4 tons per running foot, there remains for both girders 2·64 tons, being for each 1·32 tons. Table VII. shows that a wrought-iron girder of this weight will have a span of 600 to 700 feet (exactly = 666 feet).

Therefore, taking the same span for both systems, the weight of the skeleton bridge will only be half of the other, and by taking the same weight the skeleton bridge will be 200 feet longer, being about 1½ times the length of the other. If now we build a skeleton bridge of 460 feet span of steel, loaded with 10 tons per $\square$ inch, we find that each girder will weigh 0·21 tons per running foot; both girders, 0·42; roadway, 0·4; making the total weight in steel = 0·82 tons—which shows that the steel bridge will have about half the weight of the wrought-iron one, whilst the weight of the girders is in the proportion of 12 : 4; showing that a steel girder of this principle is one-third of a wrought-iron one.

LATTICE GIRDERS.

As examples for open girder bridges, we will firstly look at the Drogheda Bridge, near Dublin, in Ireland, and secondly at the Crumlin Viaduct. Both are distinguished by their light and lofty designs.

The *Drogheda Bridge* has three spans : a centre one of 267 feet, and two other ones of 141 feet, which are crossed by one continuous beam. The particulars are :—Weight of centre bridge in double line, per running foot, 1·446 tons ; or, per running foot of single line.

0·723 tons,

Greatest load = 1 ton per running foot. Strain on material 5 tons per ☐ inch in tension, and 4½ tons in compression. For 267 feet span, and 1 ton rolling load per running foot, one skeleton girder weighs 0·20 tons per running foot.

Two girders	.	.	.	0·40 tons.
Roadway .	.	.	.	0·2 "
Total weight for single line			.	0·6 "
Ditto for double line			.	1·2 "

where the weight will be identical, whether we use four or two girders. This is 0·246 tons, or ¼ ton per running foot less than the Drogheda Bridge.

The proportion will be still more favorable if we consider that all three openings are crossed by one beam, which causes the weight of the structure in the centre span to be considerably less.

The highest strain per ☐ inch was taken in this bridge 5 tons in tension and 4½ tons in compression, the tension being equal to what is calculated in our tables, and compression greater.

The *Crumlin Viaduct* will be found more suitable for direct comparison, because it represents equal spans. The particulars are : 10 spans of 150 feet each, the bridge has double lines and four girders. Weight of one pair of girders for single line = 44 tons, being 0·293 ton per running foot, and 0·147 ton for one girder. Greatest load = 1¼ tons per running foot ; greatest strain per ☐ inch = 5 tons in tension and 4 tons in compression. Deflection of one pair of girders loaded with 1¼ tons = 1½ inch.

According to our tables, a skeleton girder of the

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same span weighs 0·20 ton ; 2 girders, single line, 0·248 ton, or 2 girders of 150 feet span 37 tons ; which shows about 7 tons less for each line and each opening. Besides this we could have saved more by introducing larger spans and less pillars.

The deflection of a skeleton girder will be, at 150 feet span and 1¼ ton load, = 0·93 inches.

Finally, I shall mention a large bridge in Prussia, the *Dirschauer Weichselbrücke*, of 6 spans of 386 feet each, of which every two are crossed by one beam. The bridge is of single line, and was calculated for 0·96 ton (English measure) per running foot. The material is loaded with 4·34 tons per ☐ inch in tension, and 3·83 in compression. The dead weight is 2·873 tons per running foot.

A skeleton girder of 386 feet span and a load of 1 ton, weighs, at 5 tons strain in tension, 0·357 ton per running foot, therefore

$$\begin{aligned} \text{Two girders} \quad - \quad - \quad - \quad &= 0\text{·}714 \text{ tons.}\\ \text{Roadway} \quad - \quad - \quad - \quad - \quad &= 0\text{·}2 \quad\text{``}\\ \hline \text{Total weight} \quad - \quad - \quad 1\text{·}914 &\text{ tons :} \end{aligned}$$

i.e. a skeleton-girder bridge will weigh only one-third of the bridge over the Weichsel.

By taking the same strain of material (4·34 tons), the weight of one skeleton-girder will be 0·454 ton,

$$\begin{aligned} \text{Two girders} \quad - \quad - \quad - \quad - \quad & 0\text{·}908 \text{ tons.}\\ \text{Roadway} \quad - \quad - \quad - \quad - \quad & 0\text{·}2 \quad\text{``}\\ \hline \text{Total weight} \quad - \quad - \quad 1\text{·}108 &\text{ tons} \end{aligned}$$

being only $2\frac{3}{5}$ part of this executed bridge.

TRUSS AND BEAM BRIDGES.

As regards the remaining structures illustrated in the annexed plates, I shall firstly mention that the same calculation can be used for the trussed bridges (Fig. 2, Plate I., and Fig. 5, Plate II.) as for the suspended, only the signs of the strains will be reversed, as the system itself is reversed. The same diagrams, as shown Plate IV., therefore, obtain for the distribution of forces in a corresponding trussed bridge, under the following suppositions :—In Fig. 15 the load acts in a contrary direction, and, according to this, we have in a trussed bridge tension and compression, where in a suspended one compression and tension act respectively (*see* page 57). The red lines signify, therefore, tension, and the black ones compression. Fig. 16 is correct both for a trussed and suspended bridge ; and Fig. 17, by reversing the symbol of the colors, represents a combination of the action of load and *decrease* in temperature of 30° Centigrade.

In reference to the weights of trussed bridges I will add, that the same are not much heavier than suspended ones ; because, as the strains are the same in absolute value, such a structure can only become heavier by the increased number of struts. Fig. 15 shows, however, that for a uniformly distributed load the number of struts is not very much greater than the number of ties.

For beam bridges (Figs. 7 and 8, Plate III.) the calculation is far simpler than for the suspended, which have been examined previously. *First*, there is here nowhere a superfluous rod ; and, further, as the

girder is only fixed at the one end, and rests freely at
the other, no more points are fixed than absolutely
necessary for fixing the girder.   The calculation, there-
fore, will be carried on, as shown on pages 27, 28 ; and
there is no necessity to express the strains by the dis-
placings of the connection-points, which was the case
in the suspended bridges.   Further, it is possible at
these bridges to ascertain the forces geometrically, by
using the parallelogram of forces, but only so far as
we want the strains.   If it is, however, required to
calculate the deflection—and this is necessary to obtain
a measure of the rigidity—then we are bound to ex-
press the strains by the displacings of the connection-
points, and to find the formulæ from the displacings,
just as we did with the suspended bridges.   Besides
this, the calculation will be greatly simplified, as the
extension by heat is not resisted within the structure,
and the calculation of the action of heat is, therefore,
no longer necessary.

I consider, first, the beam (Fig. 7, Plate III.).   The
arrangement of struts and ties is here different from
the usual way, as these are not two parallel systems,
but are tending to two points.   By this, we gain the
advantage, that the bars which are only exposed to
compression are the shortest.*   It will be seen that the
bars of the greatest compression are forming the greatest
angle, and are therefore the shortest.   The bars of
greatest compression are near the abutments.   Toward
the centre, as in all similar constructions—for example,

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* In Fig. 7 the beam is drawn by mistake upside down, so that the top
ought to be the bottom.

in Warren girders—the compressive forces are decreasing, and near the centre the bars have to resist tension and compression alternately, whenever a train is passing. Corresponding to this, the bars in the centre are symmetrical, and towards the abutments the bars, which have to resist more and more compression, are becoming proportionally shorter; whilst by this, the ties become likewise proportionally longer.

For the weights of these girders I give the following table; the figures are ascertained from the medium weight of several girders which have been particularly examined, and are then calculated for various spans. (See *Appendix.*)

TABLE IX.

Containing the weight in Tons of One Girder, like Fig. 7, which can carry, besides its own weight, one Ton per running Foot.

T	25	50	75	100	150	200	250	300	350	400	500
5	1·09	4·5	10·7	20	50	100	178	300	490	800	2500
6	0·90	3·7	8·7	16	40	67	133	214	331	500	1136
7	0·77	3·2	7·4	14	33	63	106	167	250	364	735
8	0·67	2·8	6·4	12	28	53	88	136	203	285	544
9	0·59	2·4	5·6	10·2	24	45	75	115	168	235	431
10	0·53	2·2	5·0	9·1	21	40	66	100	144	200	357
11	0·48	2·0	4·5	8·2	19	36	58	88	126	174	305
12	0·44	1·8	4·1	7·5	17	32	52	79	112	154	266

This table corresponds to Table V., and according to what has been said about this one, it will not be difficult to calculate the tables corresponding to Tables VI. and VII.

Further, the following Table X. is completely analogous to Table VIII. :

TABLE X.

Containing for different materials the Spans at which a Girder, like Fig. 7, is completely loaded by its own weight.

T.	Theoretical Limit of Span.
5	600
6	720
7	840
8	960
9	1080
10	1200
11	1320
12	1440

The amount of deflection in these bridges depends chiefly on the height of the girder. The weight of the girder, however, alters only slightly by taking the heights between $\frac{1}{12}$ and $\frac{1}{4}$ of the span, because at a greater height the bracing, indeed, gets longer ; but the strains in the top and bottom member decrease remarkably.

By making, however, the height of the girder greater or smaller than this, the weight will, in both cases, increase considerably. The suitable height depends, therefore, within the given limits only on the deflection.

For iron girders, I found the deflections :—

For a height of $\frac{1}{8}$ of the span $= 0\cdot00076\ L$
"   " $\frac{1}{10}$ " $= 0\cdot00090\ L$
"   " $\frac{1}{13}$ " $= 0\cdot00110\ L,$

where L represents the span.

By taking, therefore, as is not objectionable in practice, the deflection $= \frac{1}{1000}$ of the span, we find that at a height of $\frac{1}{10}$ of the span, the girder is sufficiently rigid ; at a less height, the deflection will be too great. The weights in Table IX. are calculated for a height $= \frac{1}{10}$ of the span.

For material of greater admissible strain we are of course obliged to increase the height without obtaining, as mentioned, a much greater weight of the girder.

Fig. 8, Plate III., shows another beam. The top members form part of a parabolic curve, which might be substituted by part of a circle, and the bottom members form a straight line, as is the case in Fox and Henderson's girders. In all such parabolic girders the bracing has but little to resist, and is therefore arranged as it seemed to be most suitable and simple for practice ; and this I consider will be in making the single members in the top of equal length. I have therefore divided top and bottom into equal parts, so that the top has one bar more than the bottom. By this, the position of the bracing is completely fixed.

From these beams I have examined one, where I have taken the height $= \frac{1}{10}$ of the span. According to this calculation, I have composed the following table

of the weights, which shows that this beam is a little lighter than the previous one :

TABLE XI.

Containing the weights in Tons of One Girder, like Fig. 8, which can carry, besides its own weight, one Ton per running Foot.

T	25	50	75	100	150	200	250	300	350	400	500
5	1·00	4·1	9·7	18	49	87	152	250	395	615	1562
6	0·81	3·4	7·8	15	36	68	116	183	277	408	856
7	0·70	2·8	6·6	12	29	55	93	172	213	305	589
8	0·61	2·5	5·7	10·5	25	47	78	118	173	244	450
9	0·54	2·2	5·1	9·2	22	41	67	101	146	203	363
10	0·50	2·0	4·7	8·2	19	36	58	88	126	174	347
11	0·33	1·8	4·1	7·4	17	32	52	78	121	152	263
12	0·40	1·6	3·7	6·7	16	29	49	70	·99	135	231

This table corresponds to Table X., and the following one to Table VIII.

TABLE XII.

Containing for different materials the Spans at which a Girder, like Fig. 8, is completely loaded by its own weight.

T	Theoretical Limit of Span.
5	660
6	792
7	924
8	1056
9	1188
10	1320
11	1452
11	1584

Were we to examine the weight of these girders at different heights, we should obtain pretty near the same results as have been obtained in the previous ones; at any rate, the same would be but slightly increased, when the height is increased to $\frac{1}{5}$ or $\frac{1}{4}$ of the span; whilst also in this case the deflection increases naturally as the height increases; and reversely decreases as the height increases. For the calculated girder, I found at a height $= \frac{1}{10}$ of the span, a deflection $= 0\cdot00093$ times the span, when the girder was *completely loaded*—*i.e.* uniformly loaded, so far as admissible without exceeding the strain of 5 tons per square inch.

Which shapes for such open girders are the most economical? is a question which theory will have to decide; and whatever shape may be found to be the most economical, no difficulties can arise in making them, as it will be seen clearly skeleton structures can be carried out in any shape.

———————

As regards the different materials, which are supposed in the tables, I shall again remind you that all these tables show how very advantageous it is to adopt superior material.

The material is represented in the tables by the number T, which indicates how great a strain in tons the same can carry with safety. The different values of T, which were taken from 5 to 12 tons, represent either different material—iron or steel—or they represent to a certain degree the same material, as far as

this is more or less well made and carefully selected. Thus, for example, good iron carries with safety far more than 5 tons, as the limit of elasticity is not reached before 8 or 10 tons.   But as an iron rod, when it leaves the factory, can never be considered to be without faults or weak points, it is generally exposed to a strain of 5 tons at the utmost, and under these circumstances it would not be advisable to apply a greater strain.

If, however, every piece, before using, is submitted to a careful examination, and all the pieces are cast out which by this testing are found to be faulty, we can afterwards admit a higher strain without having less security than by using untested material.

The tables show clearly that the expenses of such a particular testing are well covered—at least in large structures, where the dead. weight, in proportion to the casual load, is considerable.

APPENDIX.

LET G be the weight of a girder, K the weight which it can carry, besides its own weight, at a uniform load —without exceeding the greatest admissible strain of material—then suppose P is the total load, including the dead weight, *i. e.*,

$$G = G + K.$$

A simple consideration shows, and the equations for the strain prove, that the following two points are correct for Skeleton Structures :

Firstly : The strain in any bar of a skeleton girder, as long as the mode of distribution is the same, is in proportion to the load P, whether this is uniformly distributed or not. Secondly : This strain is reversely proportional to the section of the bar.

By making, therefore, all sections in a skeleton girder m times as strong, all the strains will only be the mth part as before ; but by making also P m times as great, the strains will also be m times more—*i. e.* equal to their original value. The dead weight G, increases by this in the same proportion as the sections, *i. e.* G increases with P by m times its value.

As now,

$$P = G + K,$$

K must also increase at the same time as P and G, *i. e.* : If in a skeleton girder the load K increases, all sections, as well as the weight G, increase in the same proportion if the strain in all rods is to remain the same.

By this it is easy to ascertain the weight G' of a girder which is to carry the load K', if we know the weight G of a girder of the same construction carrying the load K.

It is, $$G' : G = K' : K,$$

or, $$G' = \frac{K'}{K} G. \qquad (1)$$

We need, therefore, only determine the constructions and weight of a girder for one load, in order to find directly the same for any other one.

In the same manner we can proceed when we want another strain in the rods, *i. e.* when we want to employ other material. The sections will be altered until the desired strain is received, whilst P remains unchanged, so that K increases as much as G decreases. Thus we find by formula (1), the weight G for a given K'.

It remains, therefore, now to solve the second question—How does the weight of a girder change when its length is changed, and the construction remains the same?

For this purpose we will use the following symbols :

L Length of a girder,
a Unit of length of a rod,
q Sectional area of a rod ;

the other significations remain the same as before. For a second girder, geometrically similar to the first, the same letters with a stroke (as shown above) will be used.

In altering, therefore, a into a', and L into L', we have at the same time

$$a' = n\, a, \text{ and } L' = n\, L.$$

In the same proportion we will alter the section q into q', then it is

$$q' = n\, q.$$

It now becomes the question, how must P be altered that the strains may remain unchanged?

The strain in a rod depends on the section q and on the load P of the girder : it is proportional to $\dfrac{P}{q}$ and depends on the length only so far as G changes with it, and consequently also P. Shall, therefore, the strain remain the same, $\dfrac{P}{q}$ must be unaltered, or,

$$\frac{P'}{q'} = \frac{P}{q},$$

which is only the case when also

$$P' = n\, P.$$

The dead weight G is proportional to the unit of length a and to the unit of section q, therefore to $q\, a$, and consequently from the above

$$G' = n^2\, G.$$

From all this it follows :

If without altering the geometrical figure of a girder—the length of the same, all the cross sections, and the whole load, including its dead weight which it can carry, be multiplied by n, the strain in all the bars remains unaltered, whilst the weight of the girder becomes n^2 times as great.

Assuming now we had for a girder fixed by calculation the values L'', G'', and P'', and suppose we want to find the same for another girder geometrically similar to this one, and of the length $L = n\,L''$, we find after multiplying P also by n

$$L = n\,L'',$$
$$P = n\,P'',$$
$$G = n^2\,G'',$$

therefore

$$\frac{P}{G} = \frac{P''}{n\,G''}, \qquad (2)$$

Now is, as follows by the significations :

$$P = G + K,$$

therefore $$\frac{P}{G} = 1 + \frac{K}{G};$$

further $L = n\,L''$, or $\dfrac{1}{n} = \dfrac{L''}{L}$,

therefore $$\frac{P''}{n\,G''} = \frac{P''}{G''}\,\frac{L}{L''}.$$

So that (2) becomes

$$1 + \frac{K}{G} = \frac{P''}{G''} \frac{L''}{L},$$

and if we put

$$\frac{P''}{G''} L'' = \Lambda,$$

it becomes

$$1 + \frac{R}{G} = \frac{\Lambda}{L},$$

from which we finally get

$$G = \frac{L K}{\Lambda - L} \qquad (3)$$

This formula gives us in a very simple manner the weight of a girder of the length L, which has to carry the uniformly distributed load K. The only unknown value in it is Λ, and this we can determine as soon as we have found by calculation or trial the weight G'' of a girder geometrically similar and of the length L'', i.e. that weight which the girder requires to carry, exclusive of its own weight, the uniformly distributed load R. It is:

$$\Lambda = \frac{R'' + G''}{G''} L'' = \frac{P}{G} L''.$$

From formula (3) follows that G becomes ∞ when $L = \Lambda$. The value Λ is therefore the length at which the girder can only carry its own weight without the platform and load, because its weight becomes infinitely large as soon as the load K is in value above nought. If, however, K is $= 0$, we find the corresponding value of G under the form $\frac{0}{0}$, which contains

any number, so that the girder, no matter how strong we may assume the unit of section, can always carry its own weight.   The material would, however, be exposed to the same strain as was assumed to be admissible in the first calculation.

In putting $K = L\,k,$

where k signifies the load for the unit of length, our formula gives for the whole weight of one girder

$$G = \frac{L^2\,k}{\Lambda - L},$$

and for the weight $g = \dfrac{G}{L}$ per running foot.

$$g = \frac{L\,k}{\Lambda - L}.$$

The value of Λ depends only on the greatest admissible strain, *i.e.* this value depends only on the material and the geometrical form of the girder.

In Tables VIII., X., and XII., therefore, these numbers are represented as the spans at which the girder can just carry its own weight.

THE END.

Plate1

FIG. 1.

FIG. 2.

FIG. 3.